PERFORMANCE SPECIFICATIONS OF NANOPOSITIONING SYSTEMS: ACCURACY, PRECISION, AND RESOLUTION

by

Gaurav Parmar

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Advisor:
Professor Shorya Awtar

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Abstract

A nanopositioning system is a mechatronic motion system capable of producing (moving and recording) motion with *nanometric* motion quality. Motion quality refers to the metrics accuracy, precision and resolution. The state of the art in nanopositioning systems has undergone tremendous improvements over the last two decades to achieve unprecedented levels of performance. Notwithstanding the gains that have been made, there is still a lack of consistency in the use of the terms accuracy, precision and resolution in the nanopositioning literature. There are usually two sources of confusion that arise: first, the definitions of these performance specifications, and second, the characterization procedure adopted to evaluate them. In this thesis, we discuss systematic and quantitative definitions for accuracy, precision, and resolution. A thorough review of prior art is presented, and some addendums specific to the performance specifications of nanopositioning systems are proposed. Finally, various factors that affect the motion quality of a nanopositioning system are listed and discussed. This clarification of terminology will not only help end-users objectively compare nanopositioning systems across different vendors but also allow manufacturers and researchers to better characterize their products. Although the motivation for this work comes from the research in the field of nanopositioning systems, it is equally applicable to any precision motion system in general.

Chapter 1
Introduction and Motivation

A nanopositioning system[1] is a mechatronic motion system capable of producing and recording motion with nanometric *motion quality*. We define motion quality in terms of accuracy, precision, and resolution of motion. These terms are often used by vendors and end-users to communicate the *performance specifications* of nanopositioning systems. As far as a qualitative understanding of these terms is concerned, accuracy refers to the trueness of motion, precision refers to the repeatability of motion, and resolution is often thought of as the minimum incremental motion. This qualitative understanding is shown in Fig. 1.1 [1], in which the author describes accuracy as the maximum positioning error between any two points in the machine's workspace, precision as the positioning error between a number of successive attempts to move the machine to the same position, and resolution as the smallest mechanical step the machine can make during point-to-point motion. However, there can be considerable variability in how these generic definitions are interpreted and applied. Often, different vendors use different terms to refer to the same specification. Alternately, the same term is sometimes used to imply different performance specifications by different vendors. While vendors tend to be optimistic in advertising the performance specifications of their products according to their interpretation of accuracy, precision, and resolution, the buyers in turn find it difficult to objectively compare products across different vendors. The objective of this thesis is to take a step back and revisit, and if needed reformulate, various performance specifications for the motion quality of nanopositioning systems while building on traditionally accepted terminology. This will hopefully reduce some of the confusion and misunderstanding and provide insight into various questions one should try

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[1] Also referred to as a nanopositioner or an ultra-precision positioning system

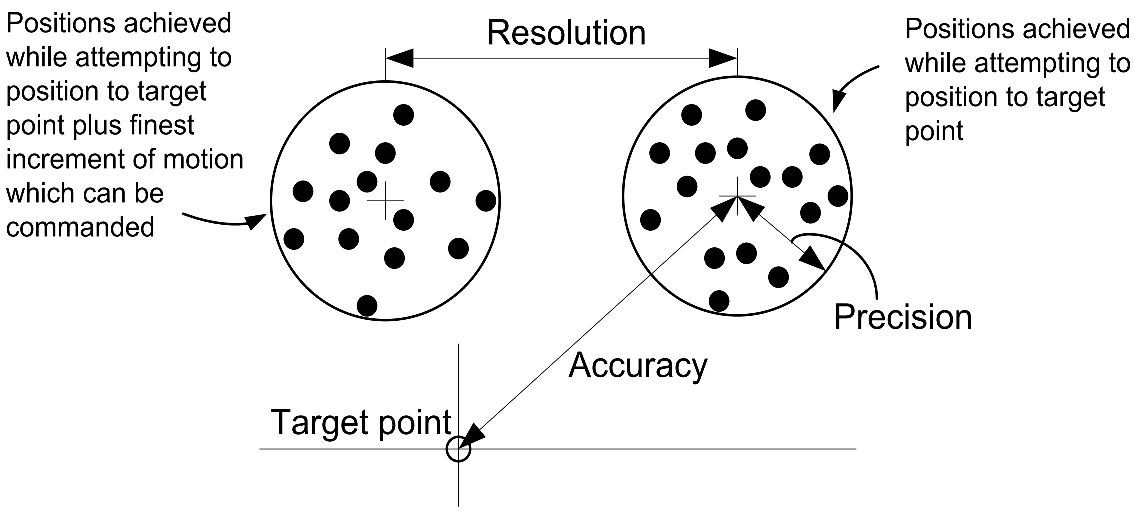

Fig. 1.1: Qualitative illustration of accuracy, precision and resolution [1]

to answer while characterizing, advertising, comparing, or buying nanopositioning systems.

Like most typical mechatronic motion systems, a nanopositioning system generally comprises a bearing that guides the motion, one or more actuators that generate the motion, drivers that operate the actuators, one or more sensors that measure the motion, signal conditioning electronics associated with the sensors, a control algorithm to meet the required motion quality, control hardware that executes the control algorithm, a power source, and often a computer-based user interface. Some nanopositioning systems may further incorporate a transmission that transmits motion from the actuator to the bearing while providing some modulation or isolation, and damping elements that help reject undesired vibrations. It should be noted that although it is the physical components and their integration that makes a nanopositioning system capable of achieving nanometric motion quality, the motion quality ultimately depends upon the closed-loop performance provided by the control system.

Due to their high motion quality, there are several existing and emerging nanotechnology applications where nanopositioners are becoming increasingly important. References [2-4] provide a good overview of numerous applications in the field of semiconductors, data storage, optoelectronics, biotechnology, nanomanufacturing, nanometrology, etc., to name a few, in which the nanopositioning system is a key enabling component. For example, as shown in the schematic in Fig. 1.2, an XY nanopositioning system forms an important subsystem of various scanning probe microscopes (SPM) such as atomic force microscopes (AFM) and scanning tunneling microscopes (STM). In these applications the nanopositioner moves the sample or the probe in a raster pattern with nanometric motion quality. The probe, mounted on a flexible cantilever, follows the surface profile, and this movement is recorded by a sensor. This measurement along with the position measurements from the nanopositioning system provides a 3-dimensional topographical image of the substrate. In addition to visualizing small features with dimensions down to size of atoms and molecules, SPMs are also used to characterize many surface-specific properties at the nanoscale such as magnetism, friction, thermal conductivity etc. [5]. Another important area of nanotechnology enabled by emergence of SPMs is that of nanomanipulation. In one such technique, commonly known as scanning probe lithography (SPL), a microscopic probe is mechanically moved across the substrate to create nanoscale features by selective deposition or removal of nano-particles, while the setup remains same as shown in Fig. 1.2 [6]. Again, a nanopositioning system with high motion quality remains a pre-requisite.

In all the above-mentioned applications, the motion quality of the nanopositioner is one the major factors that directly influences the performance attributes of the microscopy and lithography processes [7-10]. For example, the spatial resolution of the substrate image in SPM or the minimum line-width in SPL will depend on the achievable resolution of the nanopositioning system. Similarly, the lack of precision and/or accuracy of the nanopositioning system will result in a distorted image or artifact in SPM and SPL processes respectively. Therefore, it is important to define and characterize the performance specifications of nanopositioning systems without ambiguity.

Although the concepts of accuracy, precision, and resolution have been used in the context of alignment, measurement systems, and machine tools for a long time; there is still a lack of consistency in the use of these terms in context of nanopositioning systems. Only a few authors provide systematic definitions for the terms accuracy, precision, and resolution along with the procedure to evaluate these terms [11-13]. As

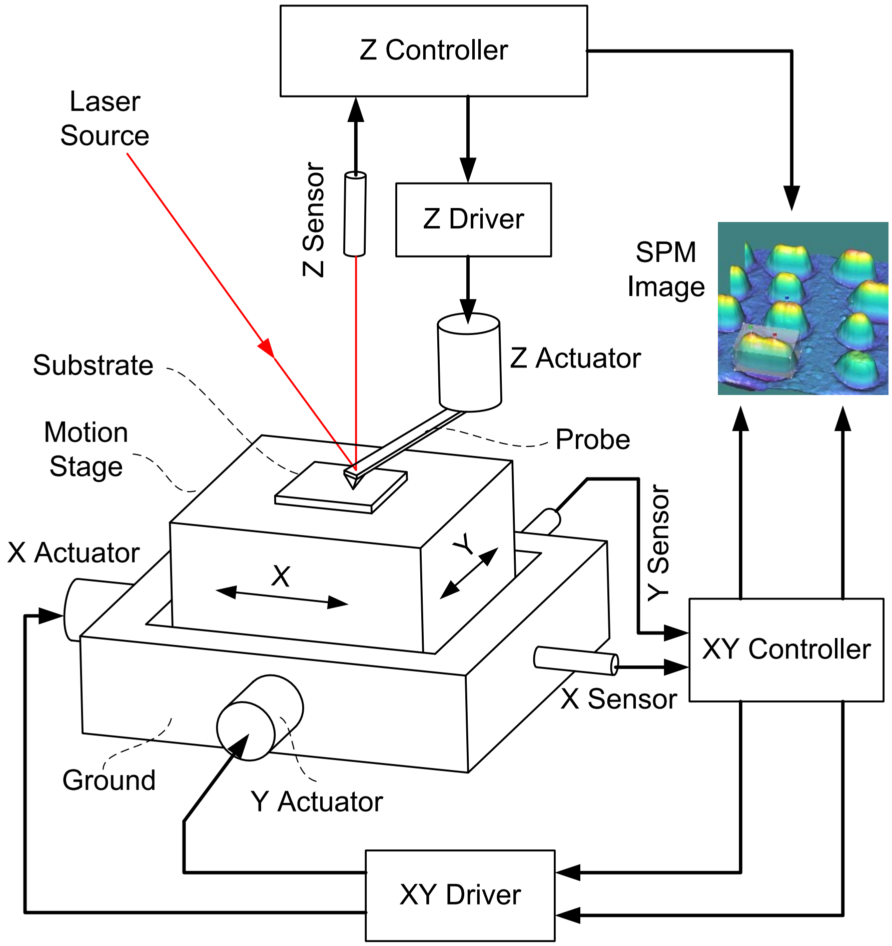

Fig. 1.2: An XY motion system employed in scanning probe microscopy

described below, there is a lack of consistency in the use of these terms in the nanopositioning literature.

There are numerous interpretations of the term motion resolution. Some characterize it in the time domain, in terms of the standard deviation or the RMS of the positioning noise [14], or using statistical tools (e.g. probability density function of the positioning noise) in the amplitude domain [11, 13] or in the frequency domain in terms of the noise floor in the power spectral density plot of the positioning noise [15]. In reference [12], the author defines resolution as the smallest step increment by which the position of the device can be consistently changed. In order to determine resolution experimentally, the command input is increased in steps and the position is measured multiple times at each step. A step is considered to be resolved if the (95%) spread in the measurements of the step does not overlap with the spread of the previous step. The resolution of the nanopositioner is the smallest measured resolution in any of the above-mentioned test.

The following comparison further illustrates the inconsistency in literature. Positioning noise has been defined as a merit for accuracy, precision and resolution by different authors. In [16], the author evaluates the precision of the nanopositioning system as the steady state positioning noise. Also, accuracy and precision have been used interchangeably. In [17], accuracy is defined as the standard deviation of the positioning error across the range of the nanopositioning system. These definitions are in contrast to most other authors, who define resolution as the smallest motion producible by the nanopositioning system referred in terms of the peak-to-peak positioning noise [13, 15].

In some cases, researchers specify RMS tracking error as a measure of the nanopositioner's performance [14, 18]. Measurements used in computing this RMS error are usually obtained using the nanopositioner's internal (feedback) sensor. On the other hand, some vendors specify accuracy and precision values for their products based on characterization using an external sensor [11, 19]. Thus, it is not clear how the performance of two such nanopositioning systems would be compared.

Many motion systems are claimed to possess nanopositioning capability simply because the resolution of their sensor is nanometric, in spite of friction in their bearings or transmission [20]. However, it is well known that because of non-deterministic effects associated with rolling/sliding interfaces such as interface tribology, friction, stiction, and backlash, it is not possible to achieve nanometric incremental steps and nanometric bi-directional repeatability [21].

Even the datasheets of the nanopositioners used by vendors to advertise their products do not provide a complete and true picture of the performance. This often makes the job of comparing products from different vendors quite difficult. Consider a buyer who is trying to compare products across different vendors in order to find a nanopositioning system that best suits his or her application. Table 1.1 at the end of this Chapter shows various performance specifications provided in the respective product's datasheet [22-27]. Most vendors do not provide quantitative definitions of the performance specifications and how these specifications are evaluated. For an interested buyer, the table raises more questions than answers. For example:

1. Do these specifications reflect the dynamic performance of the nanopositioners? Most nanopositioners are used in scanning-type path-following applications where the position command profile is dynamic in nature. For such applications, specifications derived from quasi-static tests may be inadequate and misleading.
2. Are bi-directional errors are taken into account in calculating precision? Is it sufficient to specify the mechanical backlash or hysteresis value as the precision of the system?
3. What is the minimum incremental motion that the nanopositioner can deliver reliably? If the resolution is calculated based on the positioning noise, what statistical parameters are employed to characterize the noise?
4. Are the accuracy and repeatability numbers evaluated for errors along the motion of a single-axis? If that is the case, what are the errors perpendicular to the axis of the motion? For example, in case of a single-axis system with the active axis along X-direction, what is the measure of straightness error along Y- and Z- directions?
5. How does one interpret these specifications for a multi-axis system? If the specifications are calculated along a particular axis, are these specifications also representative of the performance when two or more axes move simultaneously?
6. How are pitch, roll and yaw errors characterized for a nanopositioning system with multiple translational axes?
7. Is the accuracy value a measure of absolute accuracy or relative accuracy with respect to the commanded position?
8. Are the performance specifications evaluated from the measurements obtained from the internal (feedback) sensor or an external sensor? Specifications obtained using an internal sensor, even if traceable to a metrology standard, may not provide a true picture of the nanopositioner's accuracy.

Along with accuracy, precision and resolution, a typical datasheet of a nanopositioning system also provide other performance specifications. These include bandwidth (or speed of motion), thermal drift, load carrying capacity, operating conditions etc. The motion quality of a nanopositioning system is directly or indirectly related to the other performance specifications. For example, higher bandwidth generally leads to degradation of accuracy, precision and resolution due to excitation of resonances, and lack of disturbance rejection and command following at higher frequencies. Therefore, it is important to note that performance specifications should not be considered in isolation, and are often inter-related. However, while most performance specifications are well defined and understood, a certain level of ambiguity still lies in the understanding of accuracy, precision and resolution.

The outline of the thesis is summarized as follows: Chapter 2 provides the prior art. The terms accuracy, precision and resolution are explained in the context of a measurement system and machine tools. In Chapter 3, definition of accuracy, precision and resolution and characterization procedure along with some addendums are proposed for nanopositioning systems. Discussion in this chapter is restricted to the performance evaluation of single axis systems only along their motion direction. Definition and characterization of error terms for multi-axis nanopositioning systems is proposed in Chapter 4. In Chapter 5, various factors that affect the performance of a nanopositioner are discussed.

Table 1.1: Comparison of performance specifications across different vendors based on product datasheets

Vendor Name	Product	Accuracy	Precision	Resolution	Parasitic Motion	Ref.
nPoint	NPXY100C	Linearity Error (%)	Hysteresis (%)	Positioning Noise (nm)		[26]
Physik Intrumente	P-612.2SL	Linearity (%),	Repeatability (nm)	Open-loop Resolution (nm), Closed-loop Resolution (nm)	Pitch, Yaw (µrad)	[22]
Queensgate	NPS-XY-100A	Linearity error, Peak (%)	Hysteresis, peak-to-peak (%)	Position Noise, 1σ (nm)	Rotational Error (µrad), Othrogonality (µrad)	[23]
Mad City Labs	Nano-LR200			Resolution (nm)	Roll, Pitch , Yaw (µrad)	[27]
Piezosystem Jena	PXY100	Nonlinearity (%)	Repeatability (%)	Open-loop Resolution (nm), Closed-loop Resolution (nm)		[25]
DTI	NTS10	Accuracy (%)	Unidirectional Repeatability (nm), Bi-directional Repeatability (nm), Backlash (nm), Hysteresis(nm)	Open-loop resolution (nm), Minimum Closed-loop resolution (nm)	Pitch, Yaw (µrad)	[24]

Chapter 2
Prior Art: Definitions and Characterization

Precision, accuracy and resolution are more than century old terms used extensively in the context of measurement instruments or sensors, and motion systems like machine tools, robotic manipulators, etc. There is a significant amount of literature available on the definitions and characterization procedure of accuracy, precision and resolution in many of these applications. In this chapter, we review and summarize some of the prior art to learn how these terms have been used in various contexts such as measurement instruments, machine tools, and robotic manipulators. Some of the inconsistency and limitations in the prior art is shown. The objective is to figure out how these definitions and characterization procedures can be used and, if needed, further augmented in the context of nanopositioning systems.

2.1 Measurement Instruments

A measurement is a numerical result assigned to a physical quantity, known as measurand, with the help of a measurement instrument (or sensor) following a particular measurement process. Every measurement suffers from an error, which is defined as the difference between the value of the measurement and the true value of the measurand [28]. The process of quantifying the measurement error of a measurement instrument or a process is often termed as calibration. In other words, calibration establishes the relationship between the values of the quantities indicated by a measurement instrument and the corresponding true values of the measurand [28, 29]. In a typical calibration process, the measurements obtained from the instrument are recorded against the corresponding "true values" of the measurand over the entire measurement span or range of the instrument. Normally, the true value of the measurand is never known. However, estimates of the true value can be obtained, as described later in Section 2.1.2.

The measurement error, which is the difference between the measurement and the true measurand value, is commonly characterized in terms of accuracy, precision, and resolution. In the next three sub-sections, quantitative definitions of these three terms in the context of measurement instruments are provided, using Fig. 2.1 as an example. This figure plots the measurement error versus the true value of the measurand while the measurements are taken along the span of the instrument from both directions multiple times under similar pre-specified conditions such as temperature, pressure, humidity, etc.

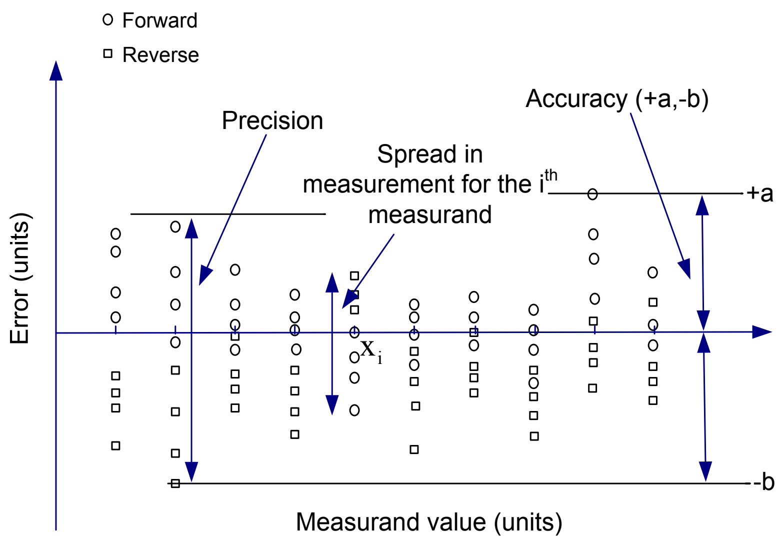

Fig. 2.1: Precision and accuracy of measurement instruments

2.1.1 Precision of Measurement Instruments

In layman terms, precision of a measurement instrument can be thought of as repeatability of its measurements. More formally, it has been defined as "*the degree of mutual agreement characteristic of independent measurements of a single quantity yielded by repeated application of the measurement process under specified conditions*" [30] and quantitatively as "*the maximum spread in the measurement of a particular measurand over several measurements from both the directions, keeping environmental conditions fixed*" [31]. The above definition of precision is referred to as "reproducibility" by the American National Standards Institute (ANSI) Standard S51.1 [32]. They make a subtle distinction between the terms "reproducibility" and "repeatability". According to this ANSI standard, repeatability is calculated as spread in measurement when a given measurand is approached from the same direction multiple times. However, both upscale and downscale readings are considered for calculation of reproducibility. Thus, in this terminology, "reproducibility" may also be referred to as "bi-directional repeatability". It may be easily seen that spread in measurement is the same as spread in measurement error. With any of these definitions, it is important to note that precision quantifies "spread" in measurement for a given measurand and not the "closeness" of a measurement to the true measurand value.

Referring to Fig. 2.1, the spread in the measurement error can be calculated at each of the measurand points. Then, the spread at any given measurand level may be interpreted as the *precision of the instrument at that measurand,* although not formally defined as such in the ANSI standard. The maximum of these spreads is defined by the ANSI standard to be the *overall precision of the measurement instrument*. In case of the spread in the measurement being random, which is often the case, the precision is quoted with a confidence level. For example, given a measurand, if 95% of the measurements are within $\pm R$ from the mean or average measurement, the precision of the measurement instrument at that measurand is reported as $\pm R$ with 95% confidence level [33].

2.1.2 Accuracy of Measurement Instruments

Accuracy is qualitatively defined as "*the closeness or the agreement between the value of the measurement and the true value of the measurand*" [30, 31]. The true value of measurand is indeterminate, since determining it would require a perfect measurement instrument and process that have zero measurement error. It may be appreciated that this is practically not possible and that every measurement instrument and process will have some finite error. However, using an instrument/process with small errors, one can get a close estimate of the true value of the measurand. In practice, the true value of the measurand is usually estimated via a measurement obtained by a reference measurement instrument that has preferably at least 10 times less measurement error than the instrument being calibrated [32, 33]. In other words, the error of the reference measurement instrument is considered small enough to be ignored [13].

ANSI standard S51.1 [32] defines accuracy of a measurement instrument, quantitatively, as *"the maximum positive and negative deviation (or measurement error) of the recorded values from the reference values (or measurand) over the range of the instrument during both upscale and downscale readings*". Referring back to Fig. 2.1, which shows measurement error vs. true value of the measurand for measurements taken along the span of the instrument from both the directions multiple times, *overall accuracy of the instrument* according to this ANSI standard can then be reported as ($+a, -b$). Along similar lines, one could interpret *the accuracy of the instrument at a given measurand* as the maximum positive and negative measurement error at that measurand, although not defined as such in the ANSI standard.

Unlike the accuracy definition provided in the abovementioned ANSI standard, some authors define accuracy in terms of bias and spread [13, 34]. This may be interpreted as a combination of the systematic error and the random error of all the measurements over the span of the instrument, where each measurand has been measured multiple times from both directions. For example, referring to Fig. 2.2, the systematic component may be specified in terms of bias or mean (m) of all of measurement error, and the random component is specified in terms of spread, quantified via the standard deviation of the error ($\pm 3\sigma$) about the mean [13, 34]. These authors term the random component or spread of the error as the "precision", which is not surprising. This definition of precision is qualitatively similar to the one presented in the previous section, in that both represent a "spread" in measurement without worrying about the bias. However, the subtle quantitative difference is that instead of looking at the spread of measurement error at each measurand separately, the spread is calculated looking at all the measurements made over the full span of the instrument. *Accuracy of the measurement instrument* is then specified as $m \pm 3\sigma$, with 99.7% confidence level assuming Gaussian distribution in the spread. The inclusion of bias in the definition of accuracy is advantageous because it originates from the systematic sources of error and therefore could be corrected (calibrated). If the bias in the measurement is corrected (calibrated), the accuracy can be just quoted as $\pm 3\sigma$. In this case, the accuracy and precision become one and the same. This definition gives us the insight that the precision of a measurement instrument determines the lowermost bound of its accuracy.

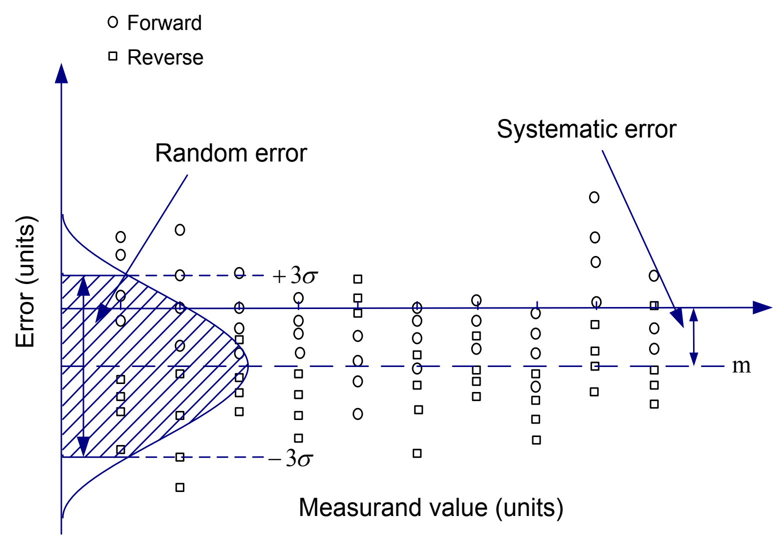

Fig. 2.2: Accuracy as a combination of systematic error and random error

In both the above-mentioned definitions of accuracy, the reference measurement is considered to be the true value of the measurand. However, there are authors who believe that the error associated with the reference measurement cannot be ignored and should rather be included in the definition of accuracy [30, 33, 35]. Thus, while the measurement error remains indeterminate, these authors define the estimate of error in terms of "uncertainty". According to Hayward [33], the *uncertainty of a measurement* is the range within which the true value of the measurand is likely to lie, at a stated level of probability. The reference further states that "*accuracy of a measurement instrument is a combination of the systematic uncertainty and random uncertainty of a typical measurement made by the instrument*". Note that this definition of accuracy is given for a particular value of the measurand and therefore may be expressed as a function the measurand along the instrument's range. To determine the *overall accuracy of this instrument* according to this definition, one would simply select the largest value of accuracy along the span of the instrument.

As mentioned already, in this definition, the total uncertainty of a measurement may be divided into two component uncertainties: random and systematic. The random component of uncertainty is characterized by the spread of the measurement values about the mean or average measurement for a given measurand. According to Hayward [29], this is the precision of the measurement instrument at a given measurand value, expressed in terms of standard deviation of the measurements. The precision of the instrument would simply be the maximum of this over the span of the instrument. This confirms with the definition of precision in ANSI S51.1 mentioned before. However, the systematic component of uncertainty, in this reference, is defined in a considerably different way compared to the bias. Consider the following example: suppose we are measuring the length of a rod and were told that its true length is exactly 1 meter, possibly based on previous measurements from a more accurate measurement instrument. Next, using a measurement instrument to be calibrated, if the average of a number of measurements is found out to be 1.005 m, then one could simply subtract the bias (1.005 – 1.000 = 0.005 m) from each subsequent measurement. Since the bias is known and can be calibrated; there is nothing uncertain about it. However, the assumption that the length of the rod is 1 m is not exactly valid. Rather, based on any previous measurement, the length would be known at best to lie within a certain range with a certain confidence level. For example, if the length of the rod is known to be within 1±0.001 m, then the bias associated with the new instrument cannot be completely eliminated. According to Hayward, one can only reduce the effect of the systematic component of the error to a certain extent. But there is bound to remain a small systematic effect of unknown magnitude which should be recognized as the systematic component of uncertainty. In the above example, the systematic component of uncertainty will be ±0.001 m. It is called a systematic component since it affects all the measurements in a similar manner. However, its actual value remains unknown. This is just one example of systematic uncertainty. In general, all the sources of systematic uncertainties in the measurement should be identified and combined to give the overall systematic uncertainty. Finally, the systematic and random components of uncertainties are combined in an RMS addition to calculate *the accuracy of the instrument* at a particular measurand.

In reference [30], Eisenhart proposes a similar procedure as above for the calculation of accuracy, i.e., by estimating the bound to the systematic and random components of uncertainties. However, he recommends that rather than performing an RMS combination, it is more logical to report the bounds on the systematic error and random error separately as it ensures better comparison of accuracy and precision between two measurement instruments.

Some authors believe that accuracy of a measurement is purely a qualitative concept and cannot be represented by a number or even a bound [35-37]. The reasoning given is that the calculation of accuracy requires one to know the true value of the measurand which is only an idealized concept. They define the bounds on the error of a measurement in terms of uncertainty. ISO Standard [28] defines the uncertainty of measurement as "*the parameter associated with the result of the measurement, that characterizes the dispersion of the values that could be reasonably attributed to the measurand*". In other words, uncertainty of a measurement characterizes the range within which the true value of the measurement is asserted to lie with a given level of confidence. Like error, uncertainty also only applies to a measurement obtained from an instrument and not to the instrument itself [33]. For the calculation of uncertainty of measurement, all the sources of uncertainties are first identified. Then, the estimate of uncertainty due to each source is estimated. Finally, the individual uncertainties are added together to arrive at the overall uncertainty of measurement. Readers are directed towards references [36, 37] for detailed guidelines and examples of calculation of uncertainty. The following example demonstrates one of the many ways noted for expressing uncertainty of measurement in the NIST guideline [36]. In this example, the nominal mass is assumed to be 100 g.

"m_s = (100.02147 ± 0.00070) g, where the number following the symbol ± is the numerical value of an expanded uncertainty $U = ku_c$, with U determined from a combined standard uncertainty (i.e., estimated standard deviation) u_c = 0.35 mg and a coverage factor k = 2. Since it can be assumed that the possible estimated values of the standard are approximately normally distributed with approximate standard deviation u_c, the unknown value of the standard is believed to lie in the interval defined by U with a level of confidence of approximately 95 percent."

2.1.3 Resolution of Measurement Instruments

Resolution of a measurement instrument (or a sensor) is defined in more than one way in the literature. Most authors [31, 34] define resolution qualitatively as the smallest measureable change in the measurand value. For a measurement instrument with a digital or quantized output, this definition may be interpreted as follows: If the input is slowly increased from a non-zero value, the output (measurement) will not change until a certain input increment is exceeded (Shown in Fig. 2.3). This could be easily appreciated using the example of a rotary incremental encoder. An encoder with 1024 pulses per revolution will have a resolution of (360/1024 =) 0.3516 degrees. This interpretation is consistent with the resolution of an instrument that provides a graded scale. The smallest gradation on the scale would represent the resolution of the graded scale.

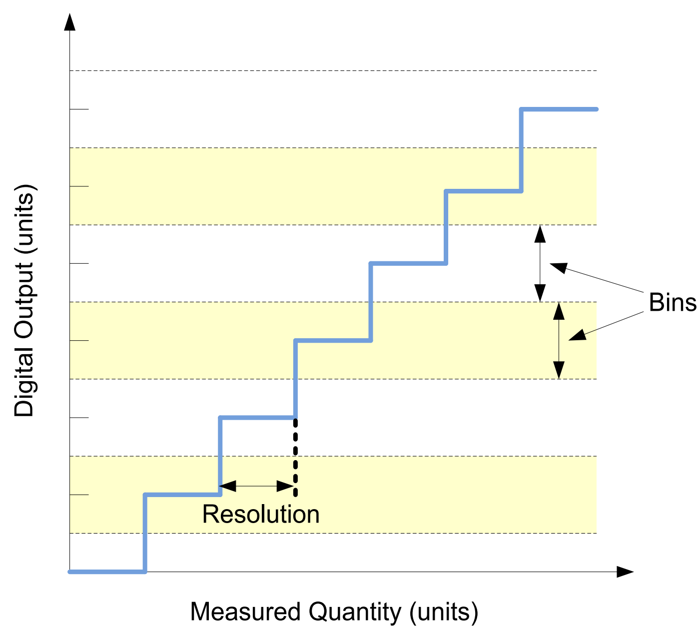

Fig. 2.3: Resolution of a digital sensor

However, the above-mentioned definition cannot be interpreted in a similar manner for a measurement instrument that provides a continuous output with no quantization. Even for a theoretically fixed value of the measurand, the output of such an instrument would exhibit some variation with time. This variation, when random, is referred to as noise and may be electrical or mechanical in nature. It can be argued that output noise is a major factor that limits the resolution of such a measurement instrument. In fact, many vendors simply mention the RMS or peak-to-peak value of the output noise to specify the resolution [38]. For an electrical output, such a specification is always be accompanied by the bandwidth of the sensor. This is because reducing the sensor bandwidth reduces the electrical noise, and hence, provides better resolution specification. However, this improvement in resolution comes at the cost of the ability of the sensor to make high frequency measurements.

Others [13, 39] define resolution as the lower bar on the difference between two measurement values which can be resolved (or differentiated) from each other with a certain confidence level. As mentioned above, in analog sensors, there is always some random noise which manifests itself as the time-domain variation in measurement. In such cases, the definition of resolution could be stated in terms of probability. In [13], the author states that "*with the assumption that the sensor noise is Gaussian with a standard deviation σ, there is 68% chance of resolving two measurands which are 2σ apart*".

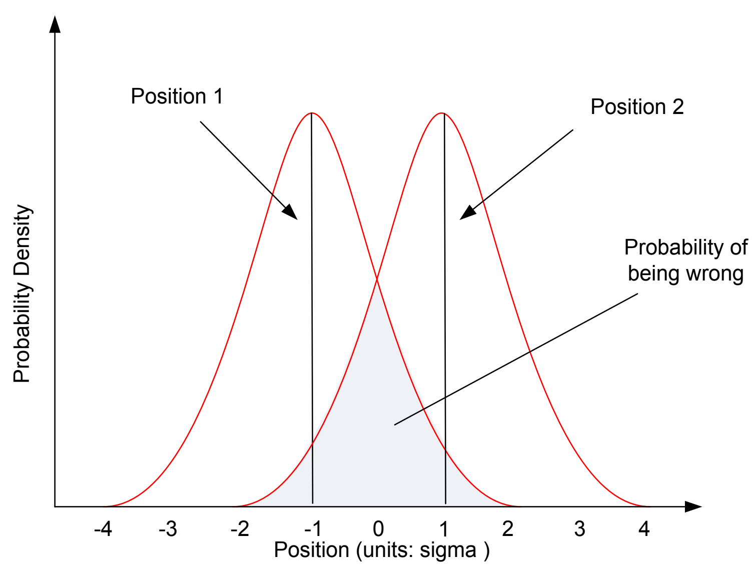

Fig. 2.4: Sensor resolution (as defined in [13])

Referring to Fig. 2.4, the author states that the Gaussian probability distribution function can be used to determine "*the probability of a single measurement being in a given region of space, and the probability of a subsequent measurement being in a neighboring region of space*". Since the author does not provide a mathematical basis for this definition, we are unable to substantiate the given definition. There can be multiple interpretations of the above statement, of which three cases were studied.

Case 1: The first measurement is smaller than the second measurement.
Case 2: The two measurements are separated by twice the standard deviation.
Case 3: Both the measurements lie in separate regions on the each side of the center of the means.

To calculate the probabilities in the abovementioned cases, consider two rods with true lengths L_1 and L_2 such that $L_2 - L_1 = d$. The sensor used to measure the length of the rods suffers from noise in its measurement. The noise is assumed to be Gaussian with standard deviation σ. Without loss of generality, the bias in the sensor could be assumed to be zero. According to Case 1, if this sensor is used to measure the length of these two rods, we could say that the rods are resolved if the second measurement (m_2) falls after the first measurement (m_1), as shown in Fig. 2.5a. This can be computed[2] in terms of probability $P(m_2 > m_1)$. This probability only depends only on the ratio of d/σ (shown in Fig. 2.6). Therefore, the probability of resolving the rods that differ by a length 2σ from each other is approximately 92%. Similarly, there is a 99.7% probability of resolving the rods if they differ by 4σ in their lengths. In other words, we could say that the resolution of the sensor is 2σ (4σ) with 92% (99.7%) confidence level.

In a similar manner the probabilities for resolving the rods in Case 2 is shown in Fig. 2.5b. This means that if the sensor is used to measure the length of these two rods, we could say that the rods are resolved if the second measurement (m_2) falls 2σ after the first measurement (m_1). This can be computed as the probability $P(m_2 > m_1 + 2\sigma)$ and is shown as a function of d/σ in Fig. 2.6.

Finally, Case 3 represents a scenario where the two rods are said to be resolved if the two measurements m_1 and m_2 lie in separate regions on the each side of the center of the means as shown in Fig. 2.5c. This can be computed as $P(m_2 > L_1 + d/2 \text{ and } m_1 < L_1 + d/2)$, and is also plotted in Fig. 2.6.

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[2] The Matlab™ code for the computation of probabilities is provided in the Appendix

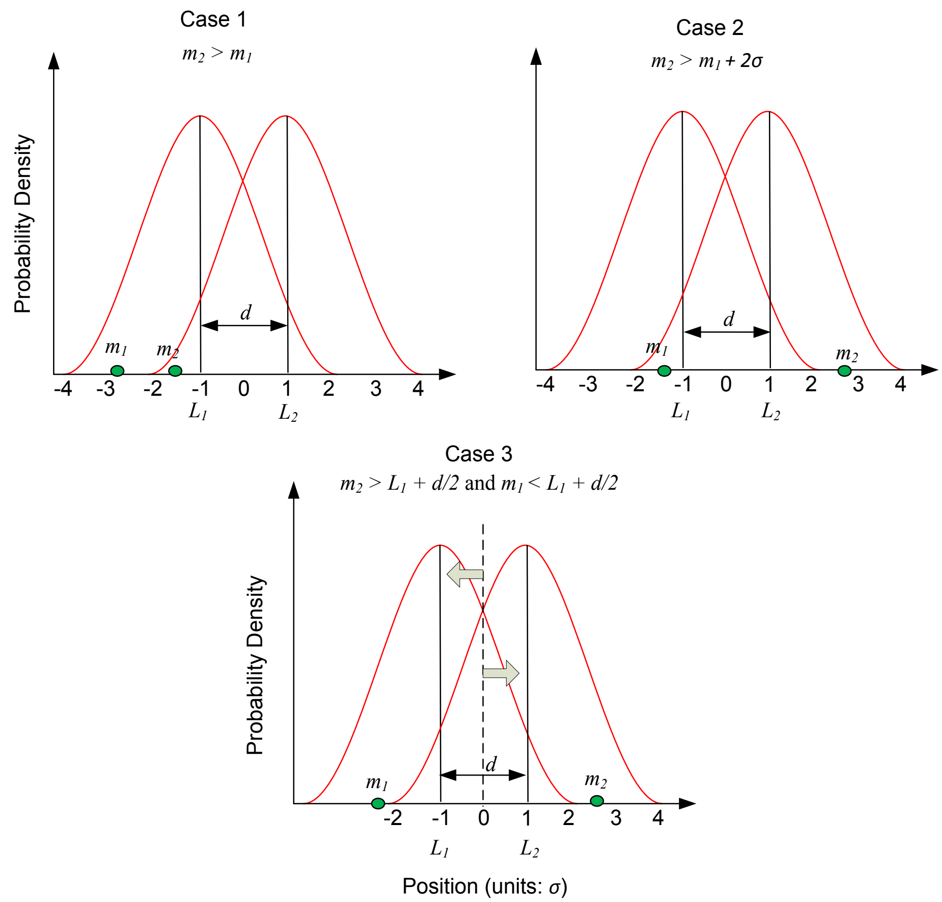

Fig. 2.5 Schematic visualization of various cases used in the definition of resolution

However, outcome of the abovementioned cases do not agrees with the author's quantitative definition of resolution in [13]. Each of the cases represents a possible definition of the term "resolution" depending upon its interpretation. Other variation might be possible to extend these definitions. For example, instead of resolving two rods as done in the abovementioned cases, one can think about resolving three such rods from each other. In such cases, the formulation of the problem and the associated probabilities will vary. As an example, let's apply Case 1 to a situation when we have to resolve three rods whose lengths are in the increments of *d*. Then, we could say that the rods are resolved if the second rod's measurement (m_2) falls after the first rod's measurement (m_1) and third rod's measurement (m_3). In this case, following the probability computation as presented above, the probability of resolving three rods that differ by a length 2σ from each other is approximately 84%. Similarly, there is a 99.6% probability of resolving three rods if they differ by 4σ in their lengths.

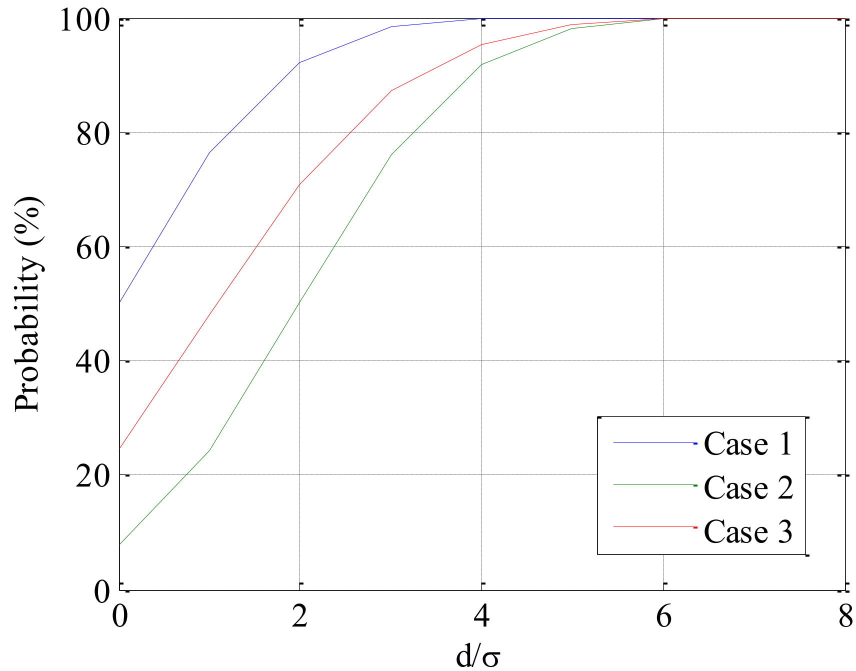

Fig. 2.6 Calculation of probabilities for various cases

Another definition for the term resolution is provided in the Measurement System Analysis (MSA) reference manual [39]. It defines "*the resolution of a measurement instrument as δ if there is an equal probability that the indicated value (measurement) of an artifact, which differs from a reference standard by less than δ, will be the same as the indicated value of the reference*". It further states the procedure for evaluating the resolution of a measurement instrument as follows: *"To make a determination in the laboratory, select several artifacts with known values over a range from close in value to far apart. Start with the two artifacts that are farthest apart and make measurements on each artifact. Then, measure the two artifacts with the second largest difference, and so forth, until two artifacts are found which repeatedly give the same result. The difference between the values of these two artifacts estimates the resolution*". This could be easily performed, for example, for a graded scale given the availability of an appropriate set of gauge blocks. However, such an experiment would be impractical in the case of a position sensor with nanometric resolution.

2.2 Machine tools and robotic manipulators

Listed below are some ways, reported in the literature, in which positioning performance is characterized in machine tools and industrial robots. These machine tools and industrial robots are generally used in manufacturing and/or assembly operations. It is important to note that all machine tools and industrial robots comprise of a motion system. The accuracy of features generated by the machine tool or industrial robot strongly depends on the accuracy of its underlying motion system, in addition to the characteristics of the process itself. The generic term "machine" will be used to encompass machine tools as well as industrial robots. One of the differences in defining accuracy and precision for motion systems as opposed to measurement instruments is in the way the error is defined. In case of measurement instruments, (measurement) error is defined as the difference between the measurement obtained and the corresponding true value of the measurand. In contrast to this, for motion systems, the (positioning) error is defined as the difference between the commanded position and the actual position attained by the stage. It should be noted that none of the references listed in this section defines resolution of a motion system.

The performance of the machine tool is affected by many error sources which may be classified as quasi-static or dynamic. While quasi-static error sources may be geometric, kinematic or thermo-mechanical in nature, the dynamic errors sources may include changing inertial loads and structural vibrations. According to reference [40], quasi-static errors account for about 70% of the total error of the machine tool and therefore have been the major focus of research in the field of machine tool's performance characterization. In the next few sections, the discussion is restricted to machine tools with translational axes for simplicity. The characterization of a machine tool with rotational axes follows a similar principle [41]. We discuss test methods and definitions provided in the standards ISO 230 titled "Test code for machine tools" for performance characterization of machine tools. ISO 230 is comprised of 9 parts which includes the following three:

Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes.
Part 4: Circular tests for numerically controlled machine tools.
Part 6: Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests).

It should be noted that, as with the case of reporting a measurement in case of measurement systems, the measured errors as well as estimated performance specifications of machine tools are reported along with the associated uncertainty [41-43]. While this is not discussed here, various references for the definition and measurement of uncertainty calculations can be found in [36, 37, 44, 45].

2.2.1 Axes related errors

The following characterization of quasi-static errors has been commonly used in the context of machine tools for more than 50 years [41, 42, 46-49]. There are six errors that can be attributed to the position of a machine tool moving along each of its motion axes: three translational and three rotational. It is important to note the distinction between the terms motion axes and machine axes. While machine axes are determined by the geometrical construction/assembly of the machine, motion axes are determined by the direction of the linear fit of the actual motion. The measurement system may be set-up with respect to either the machine axes or the motion axes. For machine tools with one or more translational axes, the translational error in the direction of motion axes is sometimes termed as the positioning error, while the translational errors perpendicular to the motion axes are referred to as straightness errors or cross-axis errors. Straightness errors do not include the linear terms [41]. The rotational errors about the motion axes are also known as pitch, roll, and yaw errors. In addition to these, the errors due to lack of perpendicularity between any two motion axes (in case of translational multi-axis systems) are also important, and are known as squareness errors [50]. Following this scheme, the total number of error terms varies depending upon the number of motion axes. For example, in the case of a 3-axis XYZ motion system, the following 21 error components could be listed [47, 49] as shown in Fig. 2.7.

a. Positioning error along each motion axis (1×3=3)
b. Cross-axis error perpendicular to each motion axis (2×3=6)
c. Pitch, roll, and yaw errors of about each motion axis (3×3=9)
d. Squareness error between any two motion axes (3)

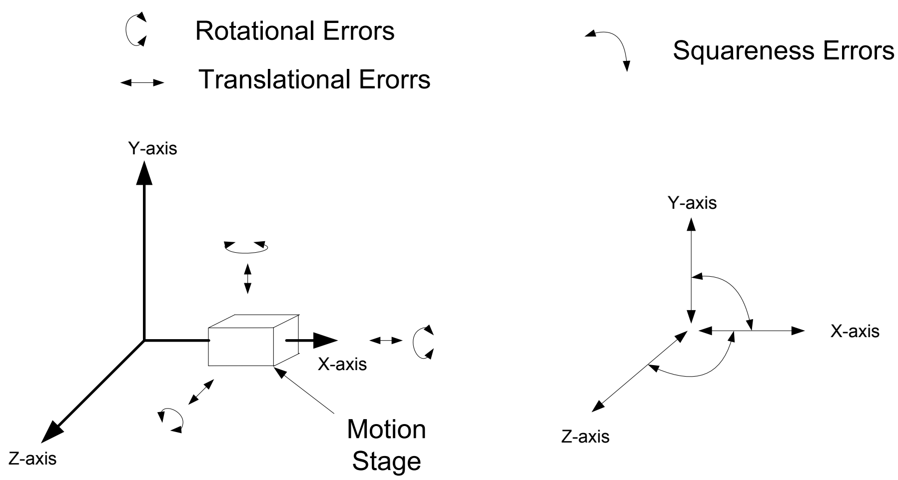

Fig. 2.7: Errors related to motion along X- (motion) axis for a 3-axis XYZ positioning system

ISO 230-2 specifies test procedures and definitions for determining the accuracy and repeatability of numerically controlled machine tools. The positioning errors are measured only along an individual axis while other axes are held stationary. The machine is moved to the predetermined target points along the axis under test from both directions. All position measurements are taken only after waiting long enough for the machine to settle down at the target position. The accuracy and precision of the machine can be then determined from the measured positioning error. This calculation is discussed in detail in Section 3.4 and 3.5. While machine tool standards like ISO 230-2 [43] and ASME B5.54 [51] specify methods for evaluating the positioning error along the direction of a particular motion axis, there are methods listed in the academic literature to evaluate all the above mentioned errors [41, 46, 48]. However, there is a disadvantage in specifying the performance of the machine tools in terms of the errors associated with individual motion axes. Many applications require motion profiles in the working space which may not be along any particular motion axis. In such cases, inertia/load changes along one axis may have a significant influence on the positioning error along other axes [41]. Furthermore, the characterization procedure followed is quasi-static in nature and thus may not include the errors due to dynamic sources such as vibration of machine structure, inertial disturbances, controller errors, etc. [7, 52, 53].

2.2.2 Volumetric error

This is defined as the positioning error at any arbitrary point in the entire working space of the system. Complete evaluation of volumetric error by measuring errors along individual motion axes is difficult and time consuming and hardly done in practice. Some standards like ISO 230-6, written for machine tools, provide guidelines for approximate evaluation of volumetric error [54]. While the characterization procedure remains similar to that described in ISO 230-2 [43], the positioning errors are not evaluated along any particular motion axis but along the body and face diagonals of the working space (see Fig. 2.8). The measurement system is set up along the body/face diagonal to be tested. The volumetric error is reported in terms of the maximum “bi-directional systematic deviation” and the maximum “reversal value” of the positioning error. The positioning error along the body diagonal depends upon the positioning errors along all the individual axes, including translational errors, rotational errors, and squareness errors [55]. However, this method, per ISO 230-6, for the measurement of volumetric errors suffers from some limitations. Translational errors are only measured along the body/face diagonals under test. Positioning errors perpendicular to the test path and orientation errors are not captured. Reference [55] provides a slight modification to above characterization process. The author shows that the body diagonal displacement measurement method is not sensitive to all angular errors. In other words, all the individual axes errors along the three motion axes are not adequately captured via the above body diagonal approach. The author suggests an alternate scheme in which the measurement system is at a slight angular offset to the motion direction along the diagonal. This way, the measured displacement error includes errors both parallel and perpendicular to the direction of motion, thereby capturing the effects of all the individual axes errors (translational as well as rotational).

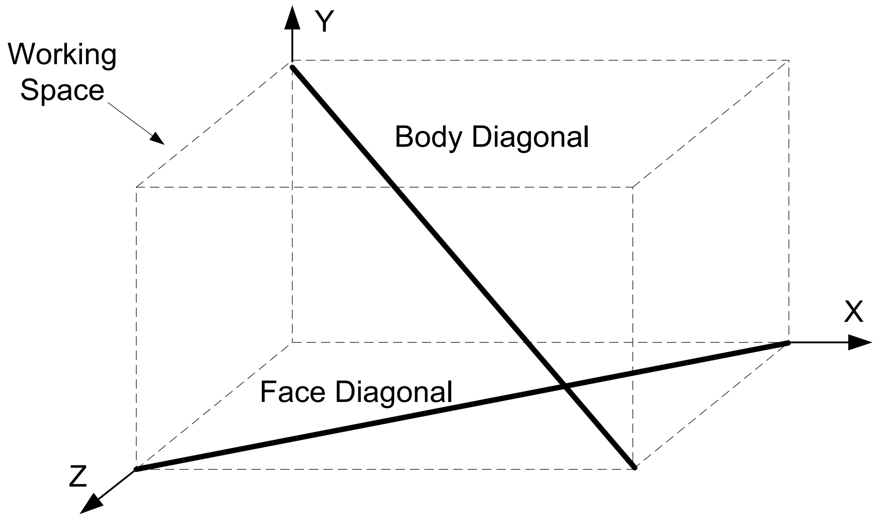

Fig. 2.8: Example of body diagonal and face diagonal in the working space of a translational 3-axis motion system

2.2.3 Path or contour related errors

ISO 230-4 provide guidelines for characterization of contouring performance when the machine tool is moved along a circular path by simultaneous movement of any two linear axes [56]. The error between the commanded circle and the actual circle is measured to evaluate: the deviation in the form with respect to the nominal (commanded) circle; the deviation of diameter from the nominal circle diameter; and the deviation of the position of the center from the center of the nominal circle. Along with capturing the influences of positioning errors along the axes, straightness errors, and squareness errors between the axes, the method also captures any motion-reversal errors and servo-mismatches present in the system [56, 57]. This provides an advantage over other standard methods like ISO 230-2 (for machine tools), which only accounts for quasi-static performance and exclude the reversal points in the error analysis. In other words, errors originating from dynamic sources are also captured. However, the estimated performance based on 230-4 is not representative of the 3D volumetric error as the test profile is confined to a plane defined by any two motion axes. Also, errors along the test path (like tracking error due to time lag) and orientation errors are not captured.

In a similar approach, ISO 9283 describes test methods for the performance evaluation of manipulating industrial robots [58]. Apart from quasi-static pose accuracy and pose repeatability, the performance specifications also include path accuracy and path repeatability. Here, pose refers to the position and the orientation of the robot. Orientation errors are also calculated around the Cartesian axes. Accuracy and repeatability are defined for both translational and orientation errors. As shown in Fig. 2.9, while errors are defined for a generalized test path in the working space of the robot, the translational errors are only evaluated in the plane perpendicular to the command path at the point of interest. Errors along the test path are not captured.

While the definitions and characterization procedure mentioned in abovementioned standards have been used extensively, it cannot be denied that they still suffer from some inconsistencies. For example, while ISO 230-2 and ISO 230-6 specify quasi-static characterization procedure, other standards such as ISO230-6 and ISO 9283 recommend dynamic test paths. ISO 230-2 and ISO 230-6 provides for the calculation of accuracy and precision along the motion direction, ISO 230-6 and ISO 9283 recommend calculating accuracy and precision perpendicular to the commanded path. Only ISO 9283 specifies the characterization procedure for rotational errors. Lastly, none of the above-listed standards define resolution for motion systems.

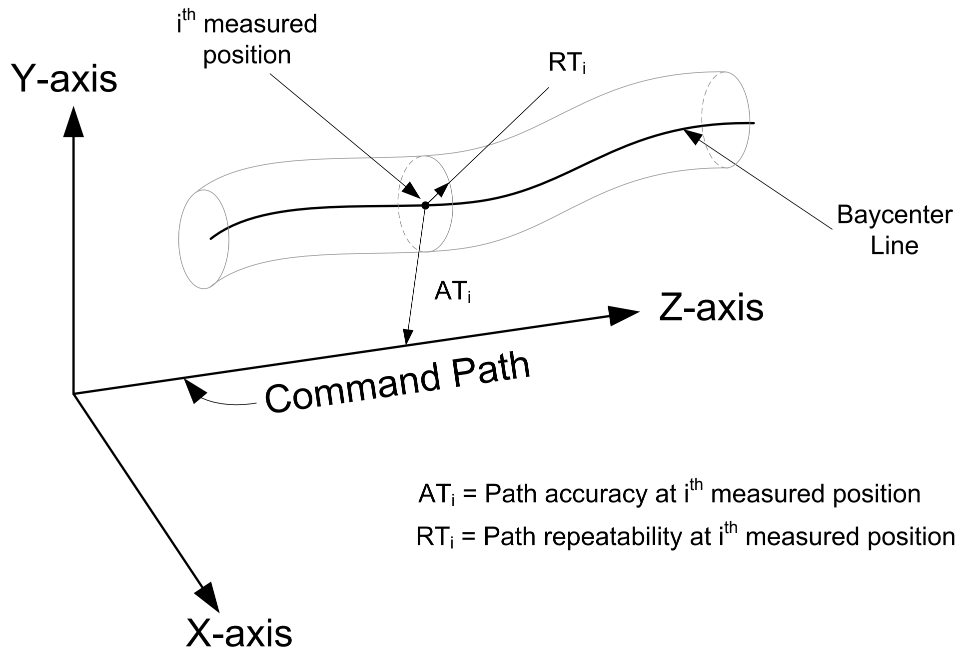

Fig. 2.9: (Translational) path accuracy and path repeatability [58]

Clearly, there is a need to standardize definitions and characterization procedure, before they could be used in the context of nanopositioning systems. In the next two chapters, we look at the definition for accuracy, precision, and resolution and characterization procedures to evaluate these performance specifications for nanopositioning systems. The emphasis is on completeness and simplicity to ensure that the recommendations can be followed consistently across industry and research labs without any confusion.

Chapter 3
Characterization of Nanopositioning Systems

Characterization of the nanopositioning system is performed to evaluate key performance specifications related to motion quality such as accuracy, precision, and resolution using a traceable length standard. This characterization can then be used to 1) Calibrate and correct for systematic errors in the nanopositioning system; 2) Enable dimensional metrology that is traceable to a known standard[3] with a given nanopositioning system, and 3) Enable an end-user to create target specifications for his/her application and provide a vendor with clear requirements.

This chapter restricts discussion to the positioning performance of nanopositioners along a single axis for simplicity. Position commands along other axes, in the case of multi-axes systems, are assumed to be kept constant during characterization. Chapter 4 treats the case where errors for multi-axis systems need to be characterized.

3.1 Internal sensor vs. external sensor

Fig. 3.1 treats a single-axis nanopositioning system as a black box. x_c represents the commanded position which can be treated as the input to the nanopositioner. x_m gives the position of the nanopositioner as viewed by the internal feedback sensor integrated with the nanopositioner. x_t is the true position of the nanopositioner as measured by an external sensor. As the true position is only an idealized concept, an external sensor that is approximately an order of magnitude more accurate (refer to the definition of accuracy of measurement systems in Section 2.1.2) than the expected accuracy of the nanopositioner is used in practice. The difference between the commanded position and

[3] Refer to Appendix A for a discussion on Traceability

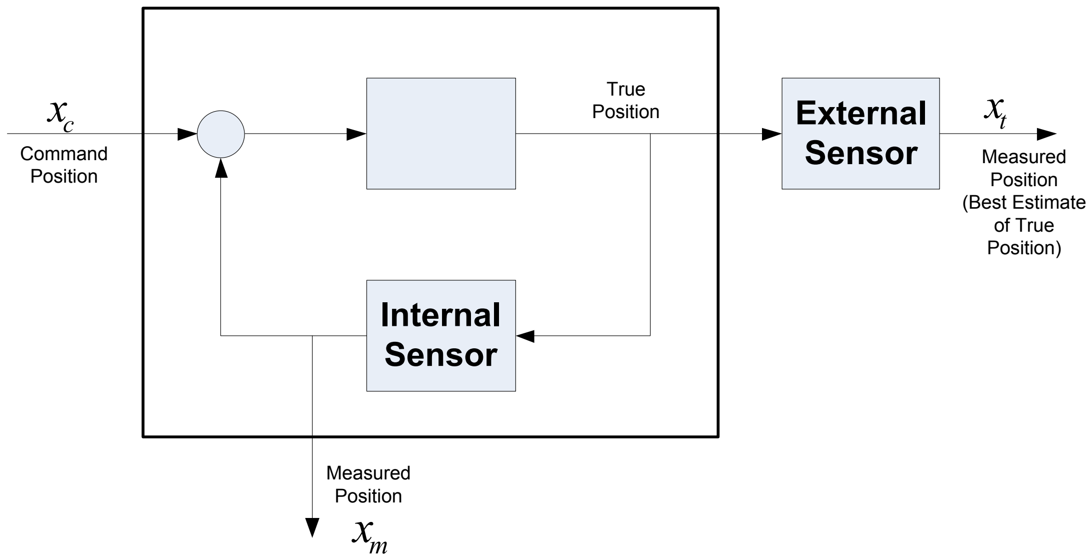

Fig. 3.1: Nanopositioning System Schematic showing internal and external sensors

the measured position is termed as the *positioning error*. The measured position may come from either the internal feedback sensor or the external sensor, depending upon the sensor used for the characterization.

Almost all nanopositioning systems currently available in industry or research labs have integrated sensor(s). The internal sensor may be used simply to keep track of the 'point of interest' or for feedback purposes. Many authors have reported performance specifications of nanopositioning systems that are based on measurements obtained using internal feedback sensors alone [12, 18, 59, 60]. Although simple, fast, and cost-effective, there are some inherent limitations associated with using the internal sensor for characterization. First, the method may lack traceability if the measurements obtained by means of the internal sensor(s) are not traceable to any length standard. Even if they are, the traceability chain may be longer which would increase the associated uncertainty. Second, this method cannot capture errors inherent to the internal sensors and associated metrology. And third, because there is no measurement of the true position, it is impossible to correct for any systematic errors that may be there in the system. ISO standard ISO 230-4 for circular tests for machine tools clearly distinguishes performance evaluation obtained using external sensors and feedback sensors respectively [56].

However, this method does capture the performance of the closed loop feedback including command tracking and disturbance/noise rejection, and thus provides some measure of the nanopositioner's positioning performance. To further explain this, consider a simple illustration shown in Fig. 3.2. Here, commanded position is the position where the motion stage is supposed to be, measured position is the position of the stage as seen by the internal feedback sensor, and the true position is the position where the stage is actually positioned [13]. It should be noted that the actual position fluctuation and sensor noise cannot be distinguished while measurements are taken via the internal sensor. Therefore, if the performance of the control system in terms of moving to a commanded position or following a desired trajectory is of concern, the method would be sufficient. However, it throws no light on the ability of the system to tell its true position to the user, which is equally important.

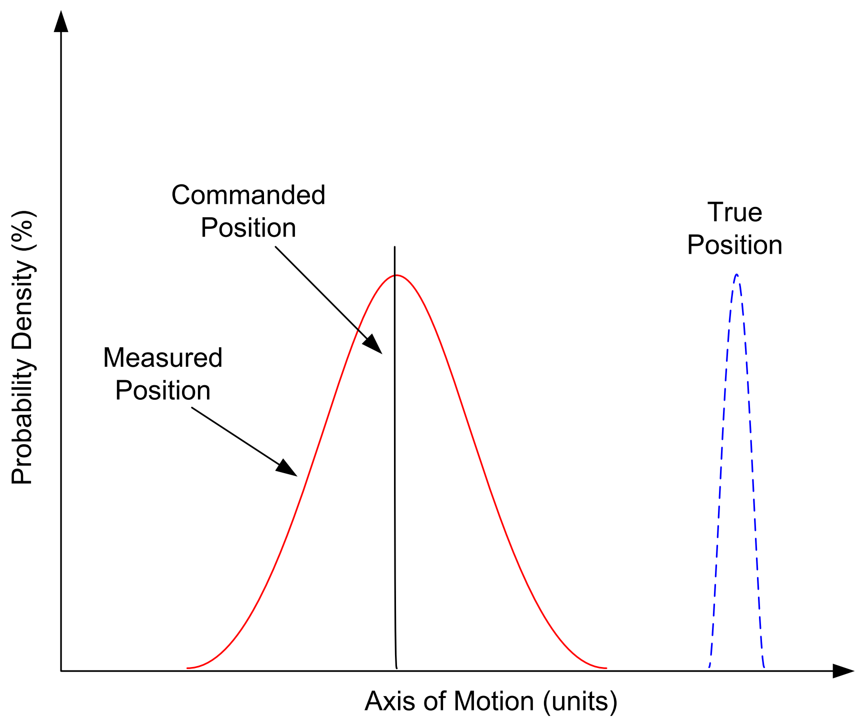

Fig. 3.2: Schematic showing commanded, measured, and true positions

The second way to perform the characterization is by using an external sensor measurement. The external sensor is not a part of the nanopositioning system and is used only for characterization purposes. Therefore, this method tends to be time consuming, costly, and may not be repeated at regular intervals if needed. However, it does overcome all the disadvantages of using the internal sensor. First, sensors such as laser interferometers, whose measurements are directly traceable to the SI unit (meter) via the wavelength of a laser [61, 62], could be used as an external sensor. This ensures a legitimate comparison of the performance specifications of different products from different vendors. Second, the method also captures errors inherent to the internal sensor and metrology setup and therefore makes it possible to correct for the systematic component of these errors. Hence, it is recommended to use an external sensor setup for the characterization purpose.

3.2 Dynamic performance vs. quasi-static performance

Various standard characterization procedures (like ISO 230-2 and ASME B5.57) are commonly used to characterize and compensate for the errors [43, 51]. Although these standards are written in the context of machine tools performance, they have also been applied for characterization of nanopositioners [63, 64]. In these tests, the system is commanded to move to a fixed number of target points along the range of motion. Measurements are taken only after the stage is allowed to stabilize at a particular position. Apart from the fact that these procedures are time consuming, they also do not provide a complete picture of system performance. This is true especially for applications where the system is expected to track a pre-defined trajectory. While these procedures have become a norm in industry and research labs, it cannot be denied that several *dynamic* sources of errors are missed by these tests [7, 52, 53]. The dynamic sources of error may arise due to any of the following reasons:

1. Inadequate command tracking: Exogenous inputs to the system may excite resonances and anti-resonances that lie outside the bandwidth of the control system. This will result in errors while tracking. Such errors cannot be captured by static calibration procedures. Also, many command profiles like triangular or saw tooth contains high frequency components that lead to positioning error during motion. This is because there is a phase lag in command tracking at higher frequencies even

for well-designed closed loop systems. Rounding of corners while following a triangular command is commonly observed [65].

2. Harmonic distortions (Noise and disturbances in the system that are function of frequency) – Some components in nanopositioning systems, for example, actuator driver or sensor driver, may exhibit harmonic distortion due to nonlinearities. Users will only observe errors due to this phenomenon when the system is excited with commands having non-zero frequency components. This will also add to error in positioning [52].

Nanopositioning systems may be used in two kinds of motion control applications: position-and-hold applications and path-following applications. The position-and-hold applications are concerned with moving the system from one point to another and staying there for some finite period of time. Only the final position is relevant and the path taken to reach that position is not. However, in the more general path-following applications, such as scanning, the system is moved along a pre-defined trajectory in time and space, and each position along this path is important. Clearly, in the case of path-following applications, the above-mentioned quasi-static characterization procedure would be insufficient. In the next section, a dynamic test cycle similar to [52, 53], used for calibration of machine tools, is proposed for characterization of nanopositioning systems as well.

3.3 Proposed dynamic test cycle

To capture errors due to various dynamic effects, measurements should be taken while the system moves along its axis continuously. With fast and powerful computers and data acquisition devices easily available, data logging and processing is hardly a concern anymore. As most of the applications require the nanopositioner to move at a constant speed [65, 66], the stage should be commanded to follow a constant velocity profile along the travel axis as shown in Fig. 3.3. Measurements are taken at pre-specified and uniformly distributed target positions along the range while the stage moves in both forward and reverse directions. Motion in both directions ensures that bi-directional effects like hysteresis and backlash are captured. The target points may be chosen to lie in a certain range of motion to avoid (or include) the errors arising from the turning action of the nanopositioning system. The measurement system should be non-contact so that it does not influence the motion of the system. It is assumed that the data acquisition rate of the measurement system is high enough that a large number of measurements can be captured on the fly. Apart from the characterization procedure, the performance of a nanopositioning system may also vary a lot with the changing environmental conditions. Therefore, to summarize, the following key parameters should be mentioned along with the characterization procedure: 1) Range of motion 2) Number of target positions 3) Motion profile and speed, and 4) Number of cycles and 5) Environmental conditions.

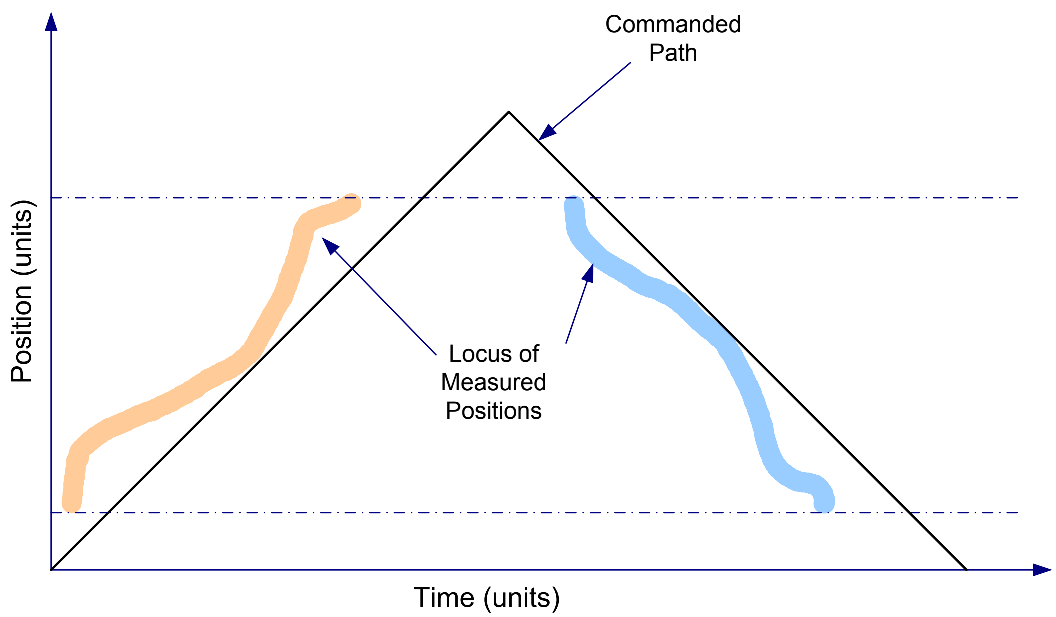

Fig.3.3: Constant velocity test profile for characterization (one cycle)

As the stage moves along the range of motion, the measured positions corresponding to the commanded (or target) positions are recorded to calculate the positioning error. Fig. 3.4 shows an illustrative plot in which the positioning error is plotted against respective target positions along the axis. The red and the blue curves denote the mean and distribution of the positioning errors in the forward and the reverse directions respectively. With a large number of target points are chosen, the curves will appear to be continuous. Once the positioning error is obtained, precision, accuracy, and resolution can be computed as described in the next few sections. The definition of precision and accuracy is taken from the standard ISO 230-2 [43]. However, the definition of accuracy is extended to make an important distinction between *absolute* accuracy and *relative* accuracy.

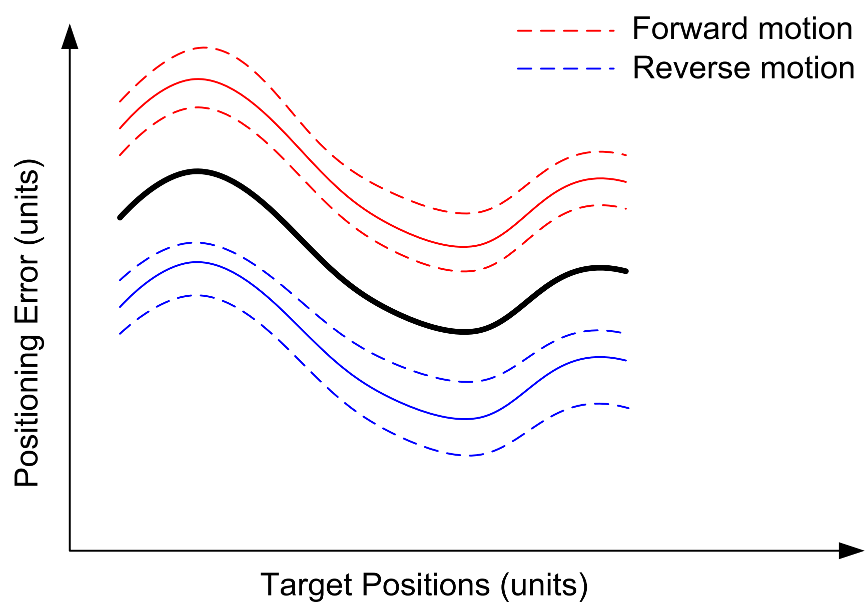

Fig. 3.4: Positioning error along the axis of motion

3.4 Precision

Precision (or bidirectional repeatability) is defined in [43] as "*the maximum value of spread in positioning error at any target point along the range of motion when the system is moved in both the directions multiple times under similar pre-specified conditions*". Hence, precision is the ability of the system to go through the same commanded position again and again from either direction under similar operating conditions.

The spread can be computed in terms of a band of ±3 standard deviations [67]. This corresponds to 99.7% confidence level assuming Gaussian distribution. But the latest standards have a slight modification [43]. First, no assumption is made about the shape of the distribution. Second, the new definition refers to the spread as the expanded uncertainty around the mean, which is 4 times the standard uncertainty (or standard deviation). Hence, a coverage factor of ±2 (instead of ±3) is used. Although all the plots are shown assuming Gaussian distribution of the spread, the same analysis holds true for any other distribution as well.

Consider the positioning error for an arbitrary target position at x_i in Fig. 3.4 obtained from the motion profile shown in Fig. 3.3. This can also be shown as a probability density function (PDF) of the positioning error (Fig. 3.5). The red and the blue curves denote the mean and distribution of positioning error at x_i in the forward and reverse directions respectively. The spread in the positioning error are marked at ±2 standard deviations from the respective means. To compute precision according to the standard ISO 230-2, first, the unidirectional repeatability is calculated at all the target points in both forward and reverse directions (denoted by $R_i\uparrow$ and $R_i\downarrow$). Then, the bidirectional repeatability at the target point x_i is calculated as shown

$$R_i = \max\left[2s_i \uparrow + 2s_i \downarrow + |B_i|; R_i \uparrow; R_i \downarrow\right] \quad (1)$$

where, $s_i\uparrow$ *and* $s_i\downarrow$ are the standard deviation of the positioning error at the target point x_i in the forward and reverse directions respectively, B_i is the difference between the mean positioning error in the forward and the reverse direction, and $R_i\uparrow = 4s_i\uparrow$ and $R_i\downarrow = 4s_i\downarrow$ denote the unidirectional repeatability in the forward and the reverse directions at the target point x_i respectively.

Finally, precision (or bidirectional repeatability) of the system along the axis is calculated as the maximum value of the bidirectional repeatability among all the target points along the axis of motion (shown in Fig. 3.5).

$$P = \max\left[R_i\right] \quad (2)$$

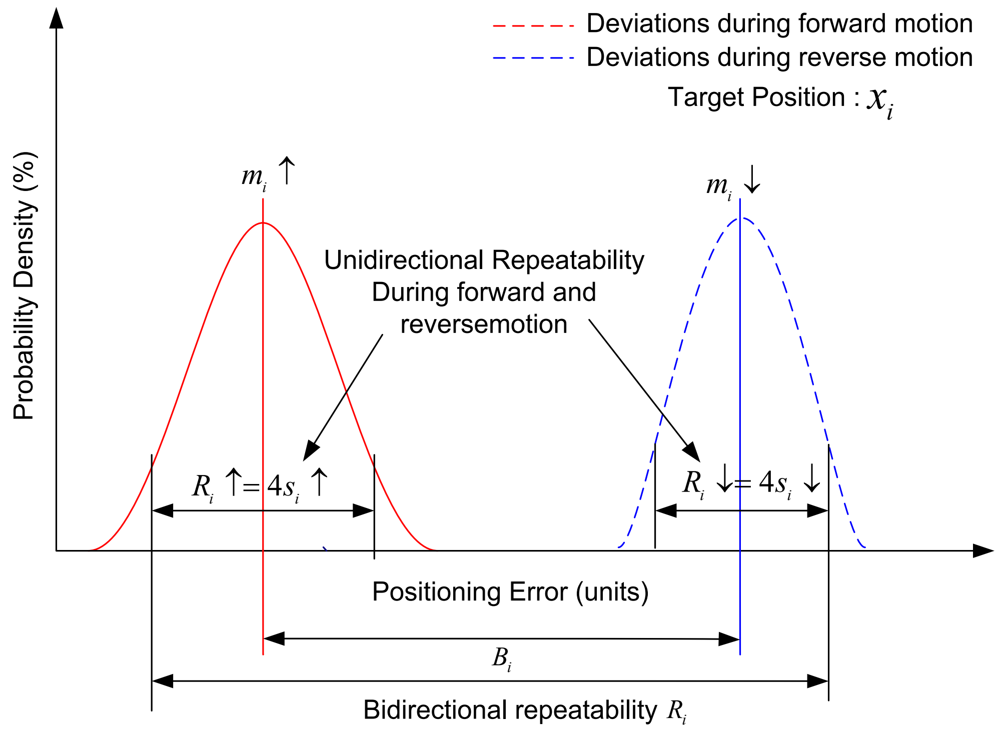

Fig. 3.5: Bi-directional repeatability at a target point x_i

3.5 Accuracy

The accuracy of a motion system may be defined in two slightly different ways depending upon the application. First, as relative accuracy if only the positioning error in the distance between any two target points is of concern, and second, as absolute accuracy if the positioning error at each target point with respect to a fixed reference is important. ISO 230-2 provides a definition of accuracy in a *relative* sense [43]. The next section describes this definition followed by a discussion on why it may not be adequate for tracking or path-following applications.

As described in ISO 230-4, (Relative) accuracy is defined as "*the maximum translational error in the distance between any two target points along the axis of motion*". Fig. 3.6 shows the PDF of the positional error at all the target points along the axis of motion as the system is moved in both directions according to the motion profile shown in Fig. 3.3. (Relative) accuracy is calculated as the difference between the maximum and the minimum positioning error along the axis of motion. The maximum and minimum error is determined by taking into account the spread of the error about the mean (which is equal to the standard uncertainty multiplied by a coverage factor of ±2). Mathematically, (relative) accuracy is calculated as follows:

$$RA = \max\left[m_i \uparrow +2s_i \uparrow; m_i \downarrow +2s_i \downarrow\right] - \min\left[m_i \uparrow -2s_i \uparrow; m_i \downarrow -2s_i \downarrow\right] \qquad (3)$$

where, $m_i\uparrow$ and $m_i\downarrow$ are the mean positioning errors at the target point x_i in the forward and reverse directions respectively, $s_i\uparrow$ *and* $s_i\downarrow$ are the standard deviation of the positioning error at the target point x_i in the forward and reverse directions respectively (refer to Fig. 3.7).

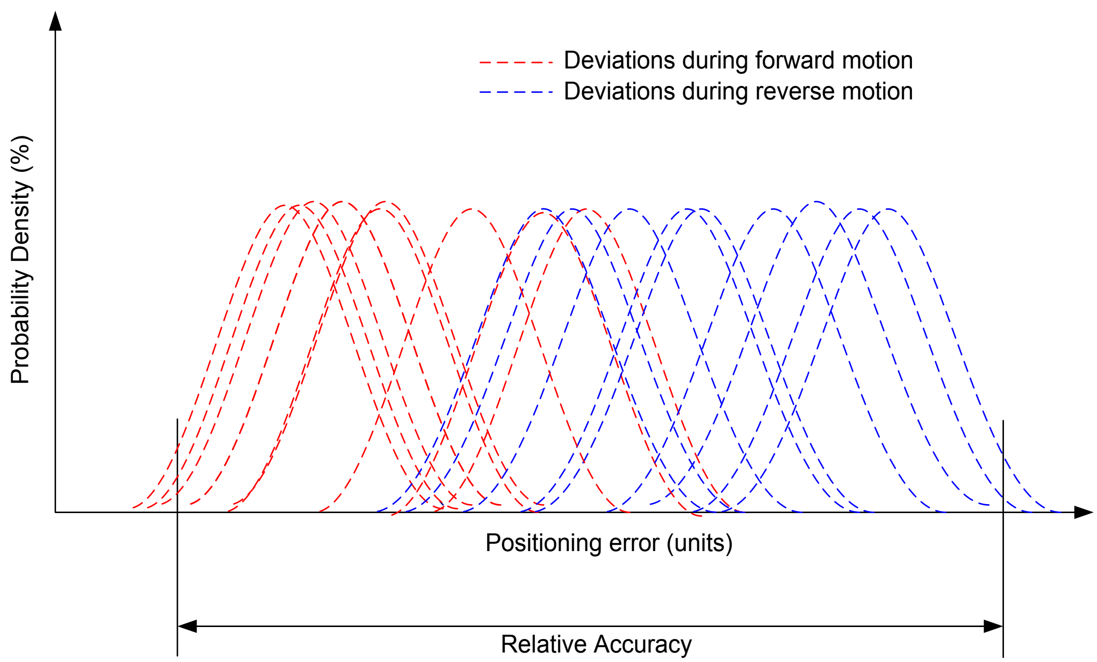

Fig. 3.6: Probability density function of the positioning error at the target points

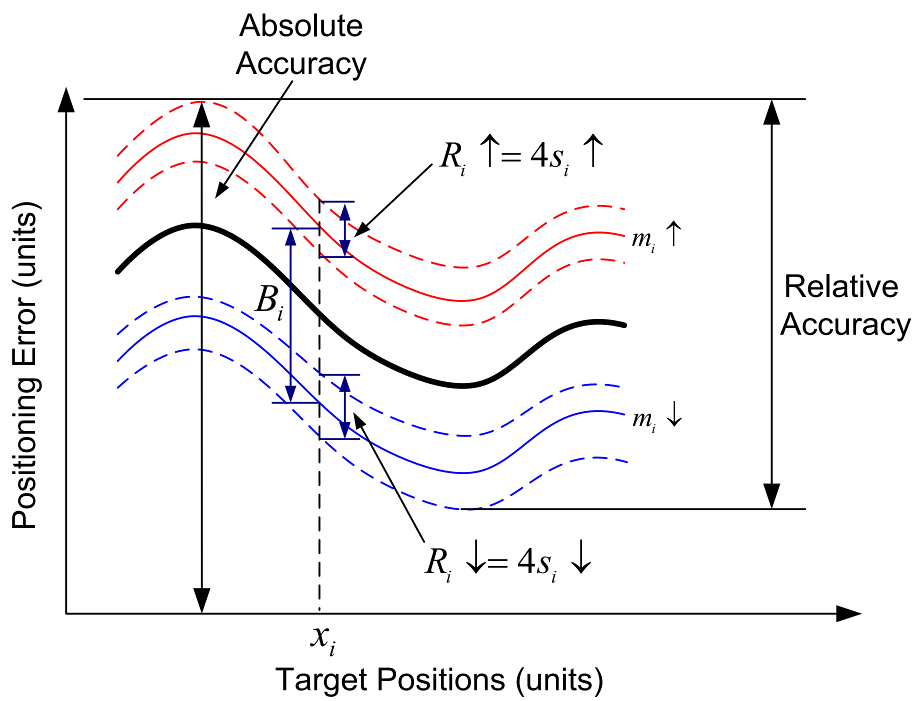

Fig. 3.7: Positioning error along the axis of motion

As seen in Fig. 3.6 and Fig. 3.7, it is important to note that the location of the spread with respect to a reference position (zero of the positioning error) is irrelevant. This definition of accuracy would suffice in certain applications; for example, when the positioning performance is gauged by measuring feature to feature distance along an axis. However, it would be inadequate for nanopositioning systems with tracking applications. The following example shows why it may not capture the true picture. Consider the bidirectional tracking performance of two different nanopositioning systems shown in Fig. 3.8. The positioning errors for the two cases are plotted against various target positions along the axis of motion in Fig. 3.9. The above definition of (relative) accuracy suggests that both systems are equally accurate but as we can see from the time domain plot, the red curve clearly demonstrates superior tracking than the blue curve.

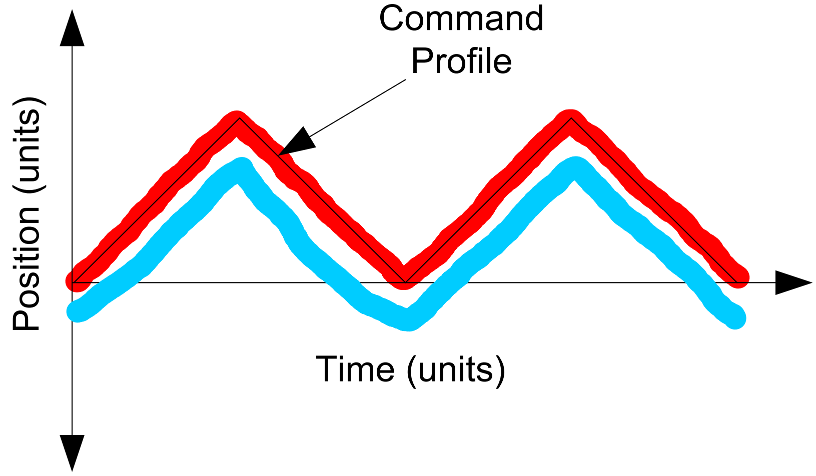

Fig. 3.8: Accuracy in context of tracking applications

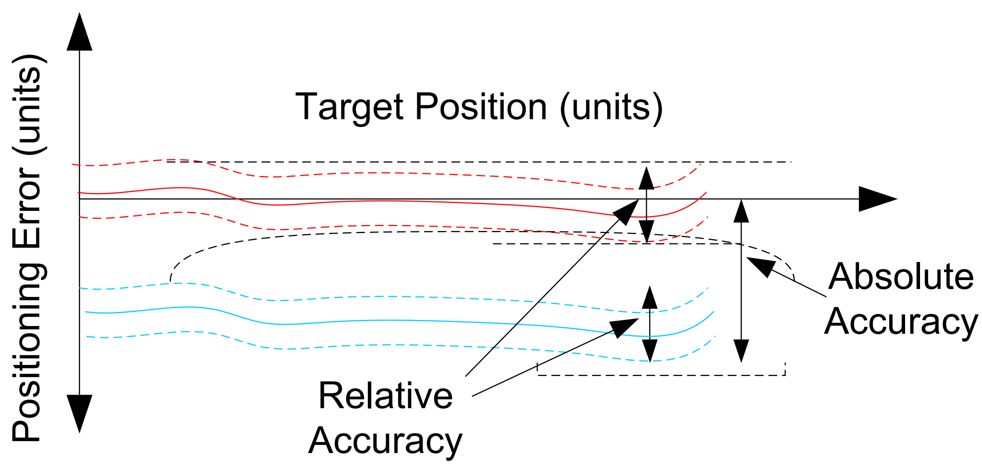

Fig. 3.9: Absolute accuracy vs. relative accuracy

To capture the tracking performance, the definition of accuracy may be modified to take in to account the offset of the spread from zero positioning error. In other words, there is a need to define accuracy in an *absolute* sense. Absolute accuracy may be defined as the combination of the maximum positive and negative positioning errors at any target point along the axis of motion. It can be calculated as follows (Fig.3.7):

$$AA = \max\left[\left|m_i \uparrow +2s_i \uparrow\right|;\left|m_i \downarrow +2s_i \downarrow\right|;\left|m_i \uparrow -2s_i \uparrow\right|;\left|m_i \downarrow -2s_i \downarrow\right|\right] \quad (4)$$

3.6 Resolution

Analogous to the definition of resolution in the context of measurement systems described in Chapter 2, resolution or minimum incremental motion of a nanopositioning system may be defined, qualitatively, as the smallest increment in the position of the motion stage such that the consecutive steps can be *resolved* (or differentiated) with a certain level of confidence. This in turn depends on the ability of the system to send a step command signal, produce a motion, and record this motion corresponding to a step change in the position. For example, the ability to command a step change depends upon the number of bits of the DAC. Bearing, actuators, and control system performance may affect the smallest increment that can be moved. Sensor and ADC affect the measurement of the position.

The absence of friction in any moving component is the foremost requirement for achieving nanometric resolution. In systems where there is no friction anywhere in the setup and DAC bit-size is not a limiting factor, there is no limitation on the ability of the system to move in small increments. In such cases, only positioning noise and resolution of a digital sensor dictate the ability to resolve any two subsequent steps. Positioning noise is the variation in the measured position with time when the motion stage is commanded to stay at any point along the axis of motion. This variation consists of the sensor's electrical noise and the actual position variations, but there is no way to distinguish one from the other.

In order to define resolution of nanopositioning systems, it is important to first understand the meaning of the term "resolved". Consider the following example shown in Fig. 3.10 to illustrate the difficulty in measuring step changes in position in the presence of positioning noise. The instantaneous position of the system is measured via a single measurement for two consecutive steps. The measurement sensor exhibits a positioning noise with a peak-to-peak variation of $2x$ (or $\pm x$). Therefore, for each measured position the true position may be anywhere in the $2x$ band around the measured position. For Case A, the measured positions differ by x. It is not even possible to ascertain whether the system has moved forward (or backward). In Case B, the measured positions differ by $2x$. Although one can be sure that the system has indeed moved forward, the actual step change in the position can be anywhere between 0 to $4x$.

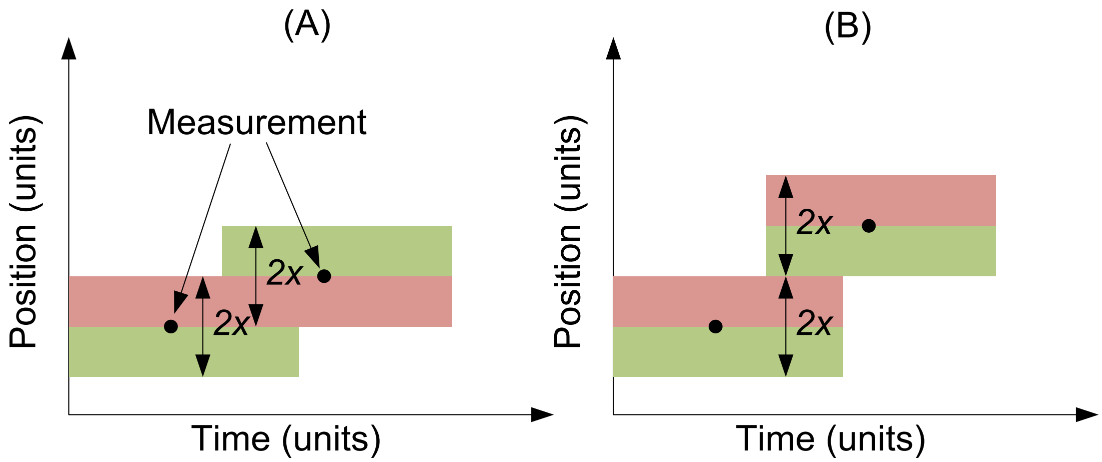

Fig. 3.10: Resolving two consecutive measurements

As seen in the prior art in Chapter 2, the term "resolved" can have multiple interpretations and corresponding varying definitions. In this section, a simpler definition based on the standard deviation of the sensor noise is presented and is proposed to be used. The thought process behind the definition is explained in a quantitative way with the help of the following measurement example. Consider 3 rods with lengths L_1, L_2, and L_3 such that L_2 differs from L_1 and L_3 by $\pm d$ respectively, as shown in Fig. 3.11. The sensor used to measure the length of the rods suffers from noise in its measurement. The noise is assumed to be Gaussian with standard deviation σ. Without loss of generality, the bias in the sensor can be assumed to be zero. Suppose the position measurements are quantized in a way that the measurements are separated in to bins of size d and the sensor indicates the value of the center of the bin. Therefore, if a measurement falls in to BIN 1, the sensor will indicate length L_1 and so on. Now, a measurement is taken to find out the length of the rod 2. The probability that the sensor will correctly indicate the length of the rod as L_2 is the measure of the resolution of the sensor. This can be computed mathematically in terms of probability $P(Indicated\ value = L_2)$. The probability depends only on the ratio of d/σ (shown in Fig. 3.12). Therefore, the probability of resolving the rods that differ by a length 2σ from each other is approximately 68%. Similarly, there is a 95.5% probability of resolving the rods if they differ by 4σ in their lengths. In other words, the resolution of the sensor is *2σ* (*4σ*) with 68% (99.5%) confidence level. Although the probability depends upon the chosen bin size, the choice of "*d*" as the bin size is intuitive as the lengths of the rods differ by the same number. Therefore, the standard and the artifact lie at the center of the bins. It should be noted that as opposed to some of the possible definitions presented in prior art, this definition is simpler as the number of rods in the though experiment remains irrelevant.

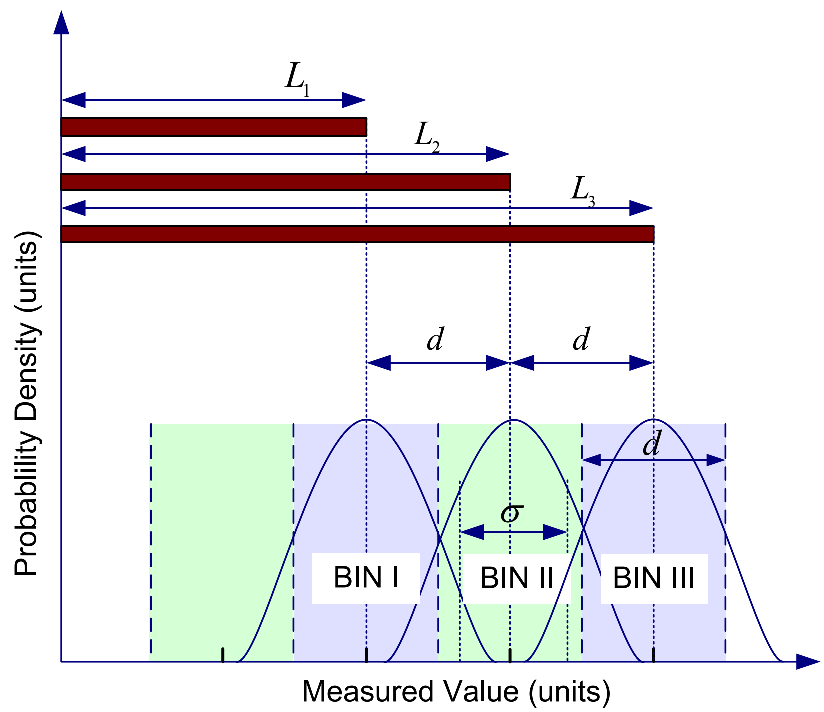

Fig. 3.11: Thought experiment to define resolving criterion

In the above explanation of sensor resolution, the sensor noise was assumed to be Gaussian in nature. In cases where the sensor noise distribution is non-Gaussian, the definition of resolution could be modified in one of the following two ways: 1) Resolution could be quoted as 2 (or 4) times the standard deviation of the sensor noise and the corresponding confidence level could be calculated. Or 2) Resolution could be quoted with 68% (or 99.5%) confidence interval and the corresponding coverage factor for a given sensor noise distribution could be calculated.

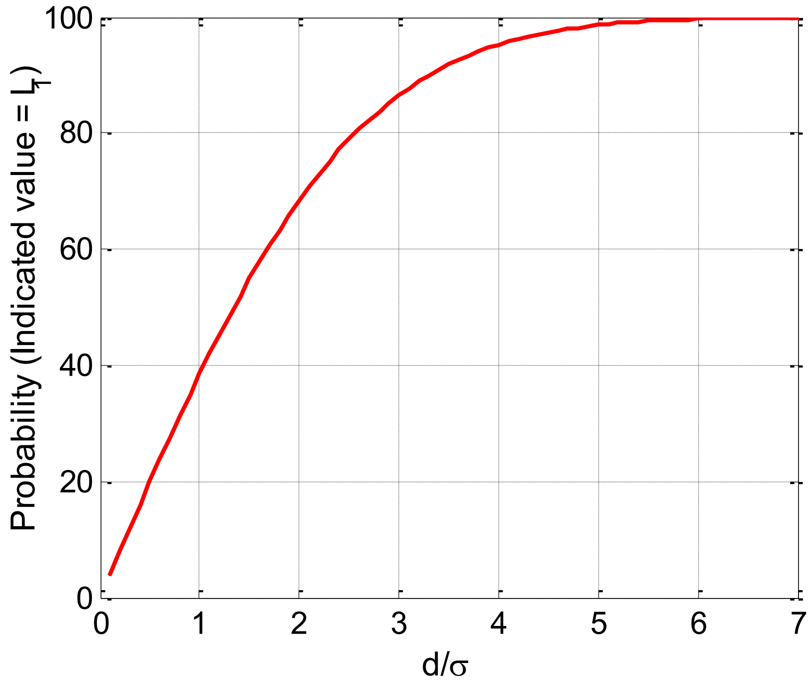

Fig. 3.12: Probability of resolving rods separated by a given distance *d* (assuming Gaussian sensor noise with standard deviation σ)

Based on the understanding of the term "resolved" discussed above, resolution of a nanopositioning system may be defined to be 2 (or 4) times the standard deviation of the positioning noise. For Gaussian distribution, this would mean that successive steps could be resolved with 65% (or 95%) confidence level.

Chapter 4

Characterization of Multi-axes Translational Nanopositioning Systems

A nanopositioning system may be classified as a single-axis or multi-axes system. In the case of multi-axes nanopositioners, the motion system configuration can be classified as either serial-kinematic or parallel-kinematic [68]. This arrangement also defines the degree of freedom and the degree of constraint directions of the nanopositioner. For example, a 2-axis XY nanopositioning system will have X and Y as degree of freedom directions and Z, Θ_X, Θ_Y and Θ_Z as degree of constraint directions. The performance measure of any nanopositioning system lies in its ability to move the motion stage along its degree of freedom directions in a controlled manner, while minimizing the motion along all other (constraint) directions. This can be further quantified in terms of the positioning error of the motion stage with respect to these directions. Based on the prior art described in Chapter 2, the following considerations are taken into account before proposing the error terms for a multi-axis nanopositioning system:

1. Evaluating the performance by measuring the positioning error of the individual axes only along their respective axis of motion is not sufficient. Straightness errors and squareness errors (for multi-axis systems) may not be neglected as compared to positioning error along the individual axes. Hence, errors should be defined and measured so as to evaluate the performance of the nanopositioning system in the entire working space.
2. Nanopositioners are often used in path-following applications where the user is interested to know the ability of the system to follow a pre-defined path. Therefore, instead of defining the translational errors in a traditional way along X-, Y-, and Z-

directions, it is more intuitive to define errors *along the path* of motion and *perpendicular to the path* of motion.

3. Machine tool standards do not incorporate orientation errors in the performance specifications. These errors are also important and directly affect many applications involving nanopositioners [9]. Hence, for a comprehensive characterization, the orientation errors must also be evaluated in addition to the translational errors mentioned above.

4. The test path should be clearly defined to evaluate the performance. For a single-axis system, a line segment along the axis could be chosen as a test path. For multi-axes systems, it is important that all the axes be moved simultaneously in order to capture cross-axis coupling and squareness errors. Therefore, for a 2-axis XY nanopositioning system, area diagonals in the working space of the nanopositioner could be chosen as the test path. Similarly, for a 3-axis XYZ system, body diagonals represent a suitable test path (Fig. 2.6). The test path used for the characterization should be explicitly mentioned in order to avoid any confusion. Also, as noted in Chapter 3, measurements should be taken "on the fly" as the system is commanded to move along the test path repeatedly in both the directions in order to evaluate the dynamic performance of the system.

If the reversal points are excluded from the analysis in case of straight line test path, errors due to turning of the axes are not captured. If these errors are of significance in the context of a particular application, a circular test path is recommended. The test circle would lie in the working plane for a 2-axis system [56]. In case of 3-axis system, a circle in any diagonal plane could be chosen (shown in Fig. 4.1). As these contours are not defined for a single axis system, motion reversal points can be included in the standard straight line test path.

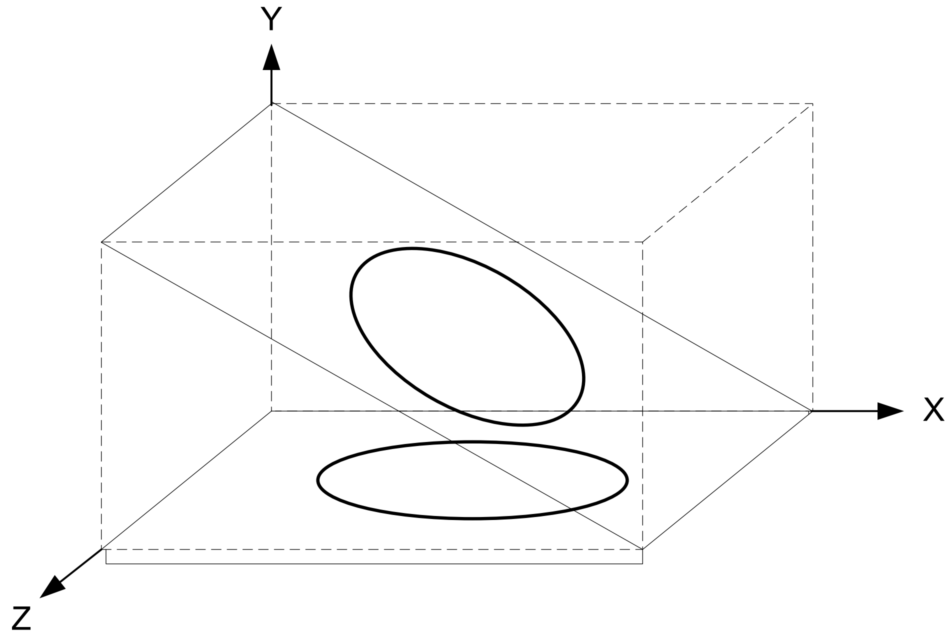

Fig. 4.1: Candidate contour test paths for a multi-axis system

In the next section, the error terms are defined for a generalized path in the workspace of a 2-axis XY nanopositioning system and therefore are applicable to above-mentioned test paths. The same definition can be extended to a 3-axis translational system. Because of the generalized form of the error definition, it is also possible for a buyer/user to ask for the performance specifications of a nanopositioning system for a pre-specified path in the context of his/her application. Once the different error terms are defined, the accuracy and precision of translational and rotational motion can be computed according to the formulae given in Chapter 3.

1. Positioning error along the path - This is defined as the error in positioning in the direction tangent to the command path at the point of interest. Consider a 2-axis XY nanopositioning system which is commanded to move along a generalized path in the XY plane as shown in Fig. 4.2. $P_c(x_c,y_c)$ is a target position on the command path and $P_m(x_m,y_m)$ is the corresponding measured position in the XY plane. The tangent to the command path at the point P_c has a slope α. In this case, the positioning error along the path (e_t) is given by

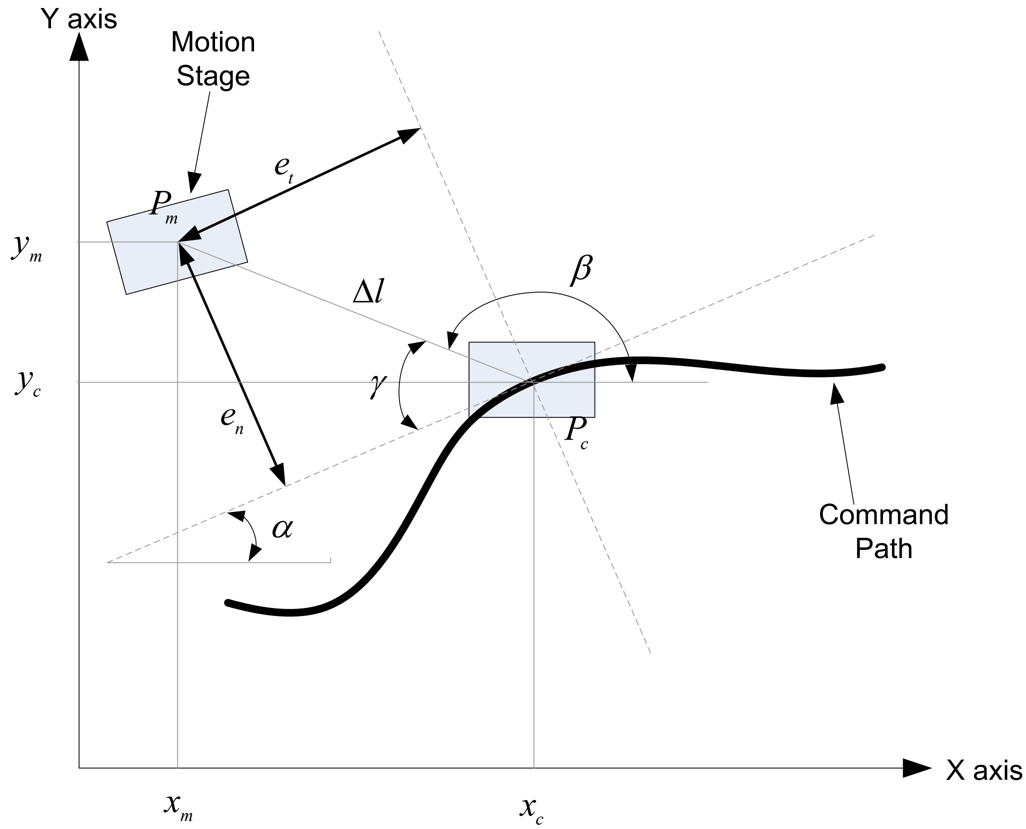

Fig. 4.2: Calculation of positioning error along the path and perpendicular to the path for a 2-axis XY system

$$e_t = \Delta l \cos \gamma \tag{5}$$

where,

$$\Delta l = \sqrt{(x_m - x_c)^2 + (y_m - y_c)^2}$$
$$, \gamma = 180^0 - \beta + \alpha \tag{6}$$
$$, \beta = \tan^{-1}\left(\frac{y_m - y_c}{x_m - x_c}\right)$$

2. Positioning error perpendicular to the path – This is defined as the error in positioning in the direction normal to the command path at the position of interest. It can be further divided into errors perpendicular to the path in the plane of the motion (e_n) and perpendicular to the plane of the motion (e_z). These errors can be computed as follows:

$$\begin{aligned} &e_n = \Delta l \sin\gamma \\ &\text{and } e_z = z_m - z_c \end{aligned} \tag{7}$$

where z_c is zero for a 2-axis XY nanopositioning system. Defining the error along these directions is advantageous as it explicitly gives the measure of positioning performance in the constraint direction (Z-direction in this case), often known as parasitic error.

It is sometimes beneficial to have a single number to describe the overall translational performance of the system in terms of its accuracy and repeatability. Vendors and users of nanopositioning system might prefer the accuracy and repeatability performance specifications to be summarized in a single number. After the accuracy and repeatability values are calculated along all the three mutually perpendicular directions, a RMS value could be computed to be referred as the overall system accuracy (A) and overall system precision (P).

$$\begin{aligned} A &= \sqrt{A_t^2 + A_n^2 + A_z^2} \\ P &= \sqrt{P_t^2 + P_n^2 + P_z^2} \end{aligned} \tag{8}$$

where A_t/P_t, A_n/P_n, and A_z/P_z are the accuracy/precision of the system along the path of the motion, perpendicular to the path of the motion (in the motion plane) and perpendicular to the motion plane respectively.

In the case of a 3-axis XYZ nanopositioning system, the two directions perpendicular to the path of the motion cannot be uniquely defined (except in cases where the test path lies in a plane and hence the above definition could be applied). In such cases, positioning error perpendicular to the path can be computed along *any* two mutually perpendicular directions in the plane perpendicular to the command path at the point of interest. After calculating the accuracy and repeatability numbers along these two directions, the accuracy and repeatability perpendicular to the path of the motion can be computed by taking an RMS as shown:

$$A_{\perp} = \sqrt{A_{\perp 1}^2 + A_{\perp 2}^2}$$
$$P_{\perp} = \sqrt{P_{\perp 1}^2 + P_{\perp 2}^2} \qquad (9)$$

where, $A_{\perp 1} / P_{\perp 1}$, and $A_{\perp 2} / P_{\perp 2}$ are the accuracy/precision of the system along two mutually perpendicular directions, perpendicular to the path of the motion.

Overall system accuracy (*A*) and overall system precision (*P*), in this case, can thus be computed as shown:

$$A = \sqrt{A_{\perp}^2 + A_t^2}$$
$$P = \sqrt{P_{\perp}^2 + P_t^2}$$

3. Orientation errors – These are defined as the angular errors in positioning about X-, Y-, and Z-axis (more generally, about the three translational directions) denoted by e_a, e_b, and e_c respectively; measured in radians or degrees. Therefore,

$$e_a = a_m - a_c$$
$$e_b = b_m - b_c \qquad (10)$$
$$e_c = c_m - c_c$$

where subscripts *c* and *m* refers to the command and measured orientations respectively. Fig. 4.3 shows the orientation error about the Z-axis for a 2-axes XY nanopositioning system. After calculating the accuracy and repeatability along these three directions, the overall rotational performance of the system can be computed by taking an RMS as shown:

$$A = \sqrt{A_a^2 + A_b^2 + A_c^2}$$
$$P = \sqrt{P_a^2 + P_b^2 + P_c^2} \quad (11)$$

where A_a/P_a, A_b/P_b, and A_c/P_c are the rotational accuracy/precision of the system about X-, Y-, and Z-axis respectively.

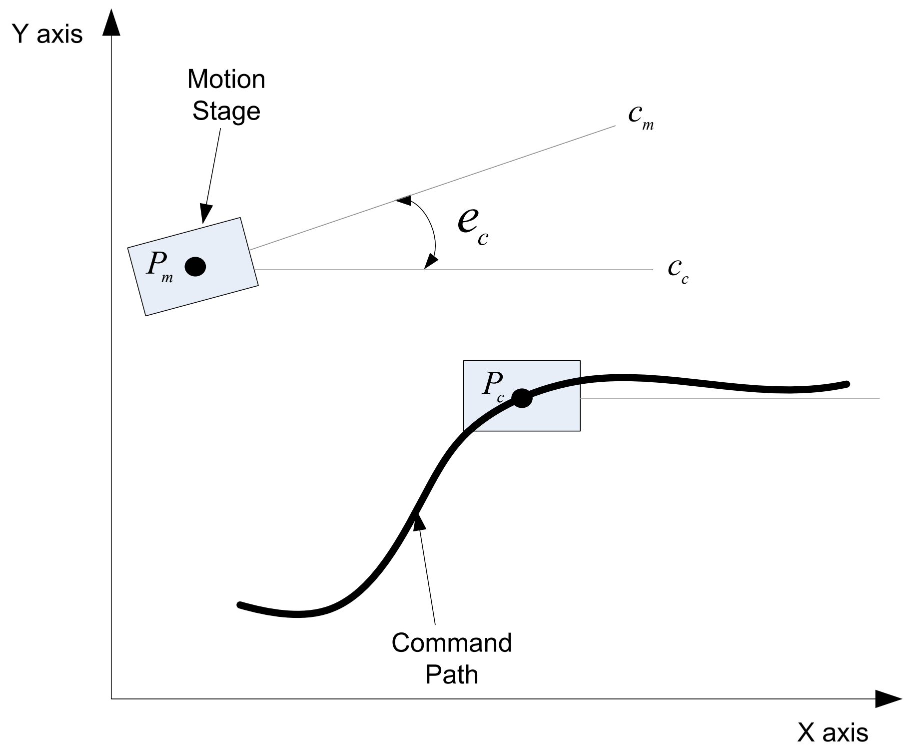

Fig. 4.3: Example of calculation of orientation error around Z-axis

In summary, the performance specifications of the system can be presented in one of the following forms as shown in Table 4.1:

Table 4.1: Presentation of accuracy and precision of a (translational) motion system

	1-axis System (X)	2-axis system (XY)	3-axis system (XYZ)
Translational Accuracy	A_t (or A_x) A_y A_z Aperpendicular Aoverall	At An Az Aperpendicular Aoverall	At An1 An2 Aperpendicular Aoverall
Translational Precision	Pt Py Pz Pperpendicular Poverall	Pt Pn Pz Pperpendicular Poverall	Pt Pn1 Pn2 Pperpendicular Poverall
Rotational Accuracy	Aa Ab Ac Aoverall		
Rotational Precision	Pa Pb Pc Poverall		

Chapter 5
Factors Affecting Motion Quality

A nanopositioning system is a combination of the physical system and the control architecture along with the environmental conditions. There are a number of factors that affects its positioning performance. As a consequence of the definitions in Chapter 3, it should be clear that factors which affect the nanopositioner's precision will also affect its accuracy. Similarly, factors which affect the nanopositioner's resolution will affect both its precision and accuracy. This relationship between accuracy, precision and resolution along with the factors that affect these performance specifications is shown in Fig. 5.1. In the next section, we briefly describe some of the major contributors that affect the positioning performance of a nanopositioning system:

1. Friction – Unlike most of their micropositioning counterparts, nanopositioning systems must operate without friction anywhere in the setup. Friction may occur in any moving component including bearings, actuators, sensors or transmission. With friction, it becomes impossible to achieve continuous motion at a nanometer level. For example, submicron level surface roughness and distortions in the bearing components may lead to discontinuous motions, which will affect the minimum incremental motion of the nanopositioner [69]. Secondly, higher values of static friction coefficient than the dynamic coefficient of friction results in stick-slip effect. This can lead to limit cycling behavior in servo, which in turn affects the motion resolution [70]. Also, linear guides with balls have backlash associated with them which could be of the order of hundreds of nanometers. Any backlash present is detrimental in achieving nanometric precision. Many commercially available nanopositioners that employs linear guides as bearings advertise nanometric resolution but suffers from relatively high precision errors [20].

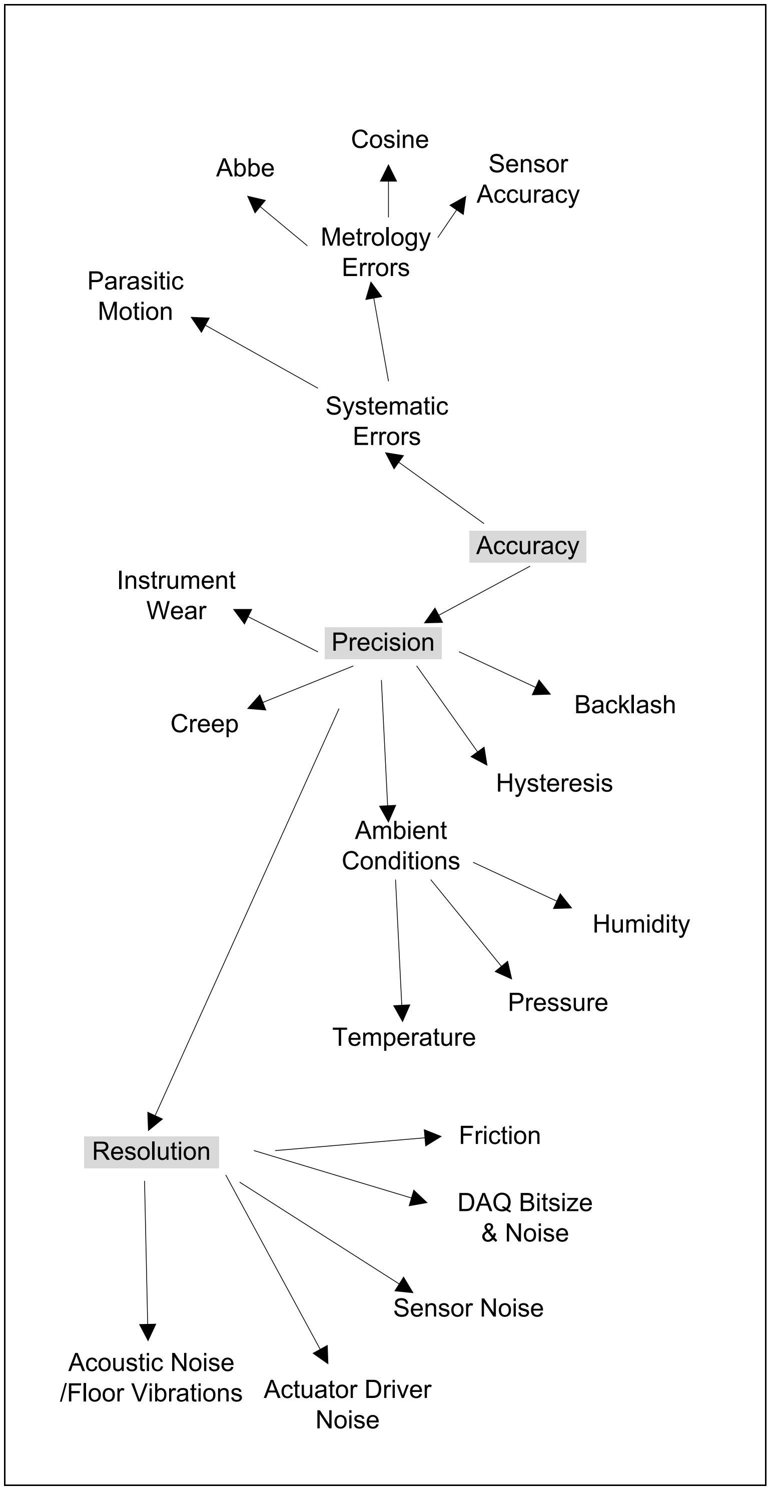

Fig. 5.1: Factors affecting motion quality

2.1. Noise – The capability to achieve nanometric resolution is largely limited by the positioning noise. It is important to understand various sources of positioning noise in the system and how they affect the performance. Fig. 5.2 shows a typical block diagram of a closed-loop nanopositioning system with various components and associated noise sources.

In general, noise sources would include feedback sensor noise, actuator amplifier noise, electronic noise in the ADC/DAC and mechanical noise or floor vibrations. Looking at the positioning noise, it is not possible to infer the contribution of these sources in the final positioning noise. This would depend upon their magnitude and where they enter the block diagram. Assuming that the all the noise sources shown are random in nature, an analysis based on the linear systems approach could be done to predict how positioning noise depends on various sources of noise in the system [71].

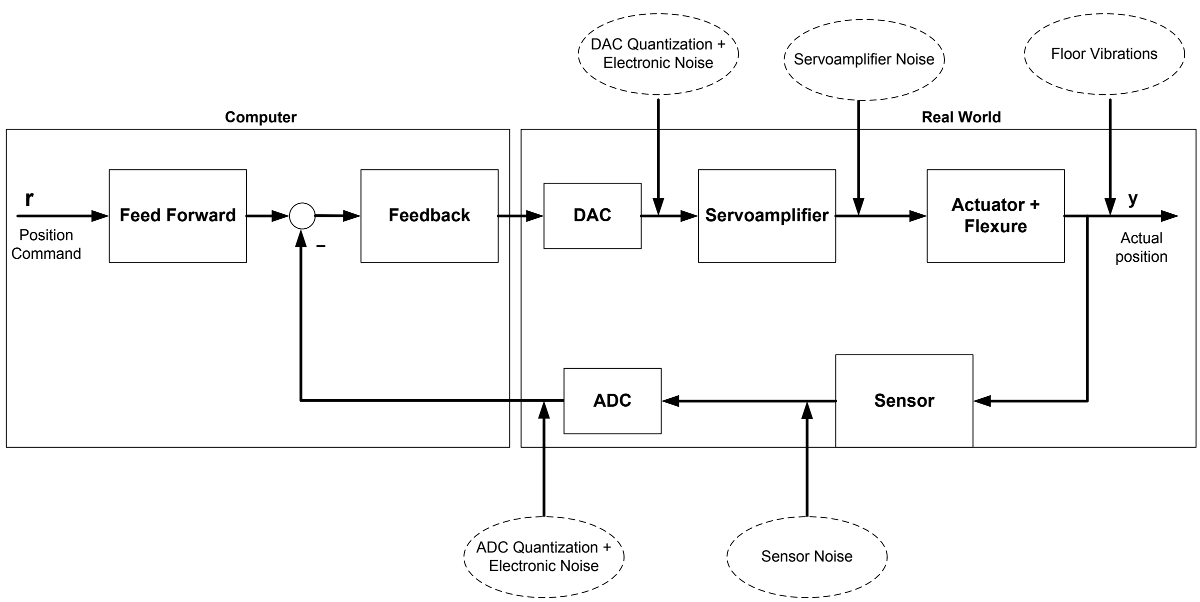

Fig. 5.2: Typical closed loop control architecture

If a random signal is passed through a linear system, the output is also a random signal. The relationship between the power spectral densities of the input and output signal is given by

$$S_y(w) = \left|H_{xy}(w)\right|^2 S_x(w) \tag{12}$$

where $H_{xy}(w)$ is the frequency response function of the linear system. Consider sensor noise for example. If we can measure the power spectral density of the sensor noise and the transfer function from the sensor noise to the final position is known, its spectral density at the position output can be easily predicted. The variance of the position signal (assuming zero mean) in absence of all other noise sources is then calculated by

$$\sigma_y^{\,2} = \int_0^\infty S_y(w)dw = \int_0^\infty \left|H_{xy}(w)\right|^2 S_x(w)dw \tag{13}$$

Finally, total variance of the positioning noise can be obtained by adding variances due to various possible noise sources.

$$\sigma_y^{\,2} = \sigma_n^{\,2}(x\%) + \sigma_{du}^{\,2}(y\%) + \sigma_{dy}^{\,2}(z\%) \tag{14}$$

The analysis above is also useful to figure out which of the noise sources has major contribution to positioning noise. For example, if the electronic noise in the actuator driver is the major component of the positioning noise, a better driver should be used, if available. Thereafter, input disturbance rejection should be given priority while designing the closed loop architecture. As an example, consider the positioning noise of a single-axis nanopositioning system currently under development. The system consists of flexure bearings actuated using a linear voice coil motor. A linear optical encoder is used for feedback purpose. Fig 5.3 shows the contributions of various sources to the final positioning noise estimated using the abovementioned procedure. The figure clearly shows that floor vibration is the biggest contributor to the positioning noise both in the open loop and closed loop.

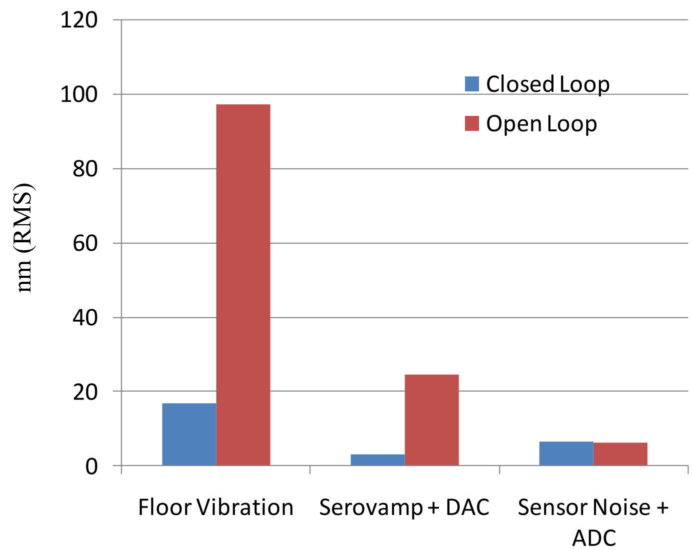

Fig. 5.3: Contribution to positioning noise in open loop and closed loop

2.2. Harmonic distortion - Apart from random noise, some sources also suffer from unwanted nonlinearities such as harmonic distortions. A distinction has to be made from random noise as harmonic distortion depends upon the amplitude and frequency of the input signal. The harmonic components would not show up in point to point positioning and hence would not affect the positioning resolution. Fig 5.4 shows the spectral power density plot of a driver for a voice coil actuator. The input to the driver is a 1V, 7Hz sine wave and the driver gain is 1A/V. The spurious components can be seen at 14Hz, 21Hz, and so on. In repetitive tracking applications where the input is some combination of sinusoidal command signals, harmonic distortions will affect the accuracy of the nanopositioner.

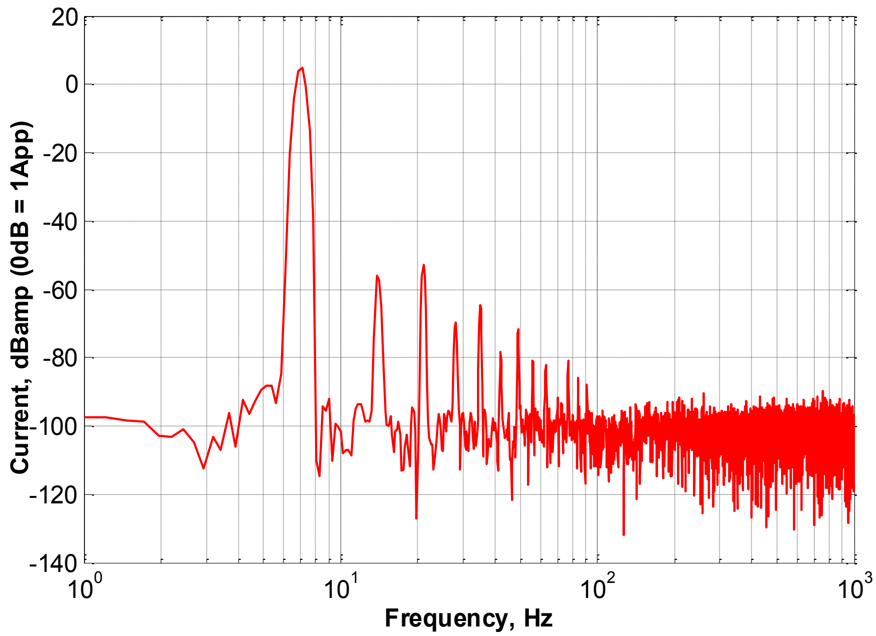

Fig. 5.4: Harmonic distortion in a voice coil actuator driver

3. Bit size - Many vendors have reported DAC Bit size as a limiting factor in achieving nanometric resolution [13, 72]. This is especially true for nanopositioners with relatively large ranges of motion. A 16 Bit DAC would divide 1mm range of motion in to 2^{16} steps, thereby providing minimum incremental steps of ~ 15 nm throughout the range of motion. Hence, the numbers of Bits of the DAC would affect the nanopositioner's capability to take minimum incremental steps. Similar arguments hold true for the Bit size of the ADC. Since the sensor signal is digitized in the ADC before the feedback loop is closed, it could also prove to be a limitation in achieving nanometric incremental motion.

4. Metrology – Assuming that the closed loop feedback is operating effectively, the measured position will be forced to be equal to the commanded position at all times. Hence, the precision and accuracy errors associated with the measurement sensor will be directly translated to the nanopositioner's performance. The same logic is true for errors caused by the metrology setup, for example, abbe error and cosine error. These errors can only be observed by evaluating the system performance against a more accurate external sensor. For example, linear optical encoders are available which can provide 1nm

resolution over long ranges, but there linearity errors can run in to few hundred nanometers [73]. On top of that, the encoder performance also critically depends upon the installation of the encoder scale and the read head.

5. Hysteresis – Hysteresis in components affect the precision of the nanopositioner. For example, PZT actuators show hysteretic nonlinearity between applied voltage and displacement which affects the bidirectional repeatability of the positioning performance. To compensate for hysteresis in the open loop, many methods have been used so far [2]. Also, most of the current nanopositioners operate in closed loop where high controller gain at low frequencies greatly minimizes the effect of hysteresis.

6. Bearing error motion – Bearing guidance errors may lead to straightness error, orthogonality errors, and parasitic motions along constraint directions. These errors may be quasi-static (kinematic) or dynamic in nature. They will also contribute towards lack of accuracy.

7. Other environmental factors – Thermal drift denotes the variations of position with change in temperature. Thermal errors occur due to the expansion properties of the materials. Even for a stage made up of Super Invar whose dimensions are of the order of ~ 100 mm, a 1^0C change in temperature would change the dimension of the stage by 30 nm. There are various sources of heat that can cause these errors including actuators, bearings, electronics and environmental changes. Environmental variations usually have less detrimental effects as the heat source is not localized. Thermal drift, although dynamic in nature, is a relatively low frequency phenomenon. Feedback control designs with large gain at low frequencies have been reported to practically eliminate creep and thermal variations [14].

Conclusion

Accuracy, precision and, resolution are commonly used to specify the performance of a nanopositioning system. However, there is a lack of consistency in the use of these terms across industry and research labs. Various sources of confusion are explained in detail. They may arise due to a) definitions of these terms and b) how these terms are evaluated in practice. Currently used characterization procedures are insufficient and may not provide a true picture of nanopositioner's performance in the entire workspace. Improved characterized procedures are proposed so that:

1. Errors which may arise due to various dynamic sources are captured.
2. Cross-coupling errors are taken in to account.
3. Disadvantages of using an internal feedback sensor vs. an external sensor are discussed.
4. Errors are defined for a generalized path in the working space of the system; they are evaluated along the path of the motion and perpendicular to the path of the motion.
5. Orientation errors are also computed.

Once the errors are calculated according to the proposed test methods, accuracy and precision can be calculated based on definitions given in ISO-230:2. However, an important distinction is made between absolute accuracy and relative accuracy. Also, qualitative definition for the term resolution is presented. Although the motivation for this work comes from the research the field of nanopositioning systems, it is equally applicable to any precision motion system in general.

In the future, these definitions and test procedures will be applied to evaluate the performance of a 2-axis XY nanopositioning system currently in development. This may provide insight in to any practical problems associated with the proposed characterization and will help in further improvement. Also, it will be worth quantifying contributions from various factors that add to the uncertainty in evaluating the above-mentioned performance specifications.

Appendix

Traceability

Traceability is the property of a result of a measurement or a value of a standard whereby it can be related to stated national or international standards through an unbroken chain of comparisons, all having stated uncertainties [28]. Traceability is the property of a measurement taken from an instrument and not of the instrument itself. Therefore, the commonly used phrase - "a NIST traceable instrument" - actually means that the measurement obtained by the instrument is traceable to the some reference standard developed by NIST through an unbroken chain of comparisons. One example of current traceability chain for nanometer scale SPM measurements to SI unit of meter is given in [8]. It is also important to note that a longer traceability chain introduces a greater level of uncertainty in the measurement.

Determination of the performance specifications of a nanopositioning system is commonly done by some characterization procedure which uses an accurate and traceable sensor, such as laser interferometer. It is the traceability of the sensor which gives the confidence to the user to compare these performance specifications for different products across different vendors, provided similar definitions and characterization procedures have been employed.

Matlab Code for Fig. 2.5

```
%% Resolution Interpretation and Calculation of
Probabilities - The code here is to calculate the
probabilities of resolving two rods using a sensor
suffering from noise in its measurement. The code mimics
the thought experiment in there representative cases. For
example, in case 1, two random measurements are obtained by
measuring two rods which are a length mu apart. The rods in
this case are considered to be resolved if the second
measurement is greater than the first measurement. The
final plot generated in the code gives the probability of
resolving the rods as a function of d/σ, where d is the
difference in the length of the two rods and σ is the
standard deviation of the sensor noise.


%% Constants
sig = 1;
mu = sig*[0:1:8];
n = 50000;

%% Case I : Probability that m1 < m2
for j = 1:length(mu)
    count = 0;
    rng('shuffle'); m1 = sig*randn(n,1); % First
measurement
    rng('shuffle'); m2 = mu(j) + sig*randn(n,1); % Second
measurement
    for i = 1:n
        if m1(i) <= m2(i) % Case I
            count = count + 1;
        end
    end
    p1(j) = 100*count/n; % Probability for case I
end

%% Case II : Probability that m1 + 2*sigma < m2
for j = 1:length(mu)
    count = 0;
    rng('shuffle'); m1 = sig*randn(n,1);
    rng('shuffle'); m2 = mu(j) + sig*randn(n,1);
```

```
    for i = 1:n
        if m1(i) + 2*sig <= m2(i)
            count = count + 1;
        end
    end
    p2(j) = 100*count/n;
end

%% Case III : Probability that m1 < d/2 and m2 > d/2
for j = 1:length(mu)
    count = 0;
    rng('shuffle'); m1 = sig*randn(n,1);
    rng('shuffle'); m2 = mu(j) + sig*randn(n,1);
    for i = 1:n
        if m1(i) <= mu(j)/2 && m2(i) >= mu(j)/2
            count = count + 1;
        end
    end
    p3(j) = 100*count/n;
end

plot(mu/sig,p1,mu/sig,p2,mu/sig,p3); grid on; hold on;
ylabel('Probability (%)');
xlabel('d/\sigma');
xlabel('Case 1','Case 2','Case 3');
```

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