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Mechatronic design of a ball-on-plate balancing system

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Abstract

This paper discusses the conception and development of a ball-on-plate balancing system based on mechatronic design principles. Realization of the design is achieved with the simultaneous consideration towards constraints like cost, performance, functionality, extendibility, and educational merit. A complete dynamic system investigation for the ball-on-plate system is presented in this paper. This includes hardware design, sensor and actuator selection, system modeling, parameter identification, controller design and experimental testing. The system was designed and built by students as part of the course *Mechatronics System Design* at Rensselaer. © 2002 Elsevier Science Ltd. All rights reserved.

1. Mechatronics at Rensselaer

Mechatronics is the *synergistic* combination of mechanical engineering, electronics, control systems, and computers. The key element in mechatronics is the *integration* of these areas through the design process. The essential characteristic of a mechatronics engineer and the key to success in mechatronics is a *balance* between two sets of skills: modeling/analysis skills and experimentation/hardware implementation skills. Synergism and integration in design set a *mechatronic system* apart from a traditional, multidisciplinary system. Mechanical engineers are expected to design with synergy and integration and professors must now teach design accordingly.

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Nomenclature

m_b	mass of ball
r_b	radius of ball
I_b	inertia of ball about its own center
I_{xx}	inertia of plate about its x-axis with respect to point O
I_{yy}	inertia of plate about its y-axis with respect to point O
h	offset distance of the plate from O
q_1, q_2	plate angles
θ_{m1}, θ_{m2}	motor angles
(x_b, y_b)	ball position with respect to the plate

In the Department of Mechanical Engineering, Aeronautical Engineering and Mechanics at Rensselaer there are presently two senior-elective courses in the field of mechatronics, which are also open to graduate students: *Mechatronics*, offered in the fall semester, and *Mechatronic System Design*, offered in the spring semester. In both courses, emphasis is placed on a balance between physical understanding and mathematical formalities. The key areas of study covered in both courses are:

1. Mechatronic system design principles.
2. Modeling, analysis, and control of dynamic physical systems.
3. Selection and interfacing of sensors, actuators, and microcontrollers.
4. Analog and digital control electronics.
5. Real-time programming for control.

Mechatronics covers the fundamentals in these areas through integrated lectures and laboratory exercises, while *Mechatronic System Design* focuses on the application and extension of the fundamentals through a design, build, and test experience. Throughout the coverage, the focus is kept on the role of the key mechatronic areas of study in the overall design process and how these key areas are integrated into a successful mechatronic system design.

In mechatronics, balance is paramount. The essential characteristic of a mechatronics engineer and the key to success in mechatronics is a balance between two skill sets:

1. Modeling (physical and mathematical), analysis (closed-form and numerical simulation), and control design (analog and digital) of dynamic physical systems.
2. Experimental validation of models and analysis (for computer simulation without experimental verification is at best questionable, and at worst useless), and an understanding of the key issues in hardware implementation of designs.

Fig. 1 shows a diagram of the procedure for a *dynamic system investigation* which emphasizes this balance. This diagram serves as a guide for the study of the various mechatronic hardware systems in the courses taught at Rensselaer. When students perform a complete dynamic system investigation of a mechatronic system, they develop modeling/analysis skills and obtain knowledge of and experience with a wide variety of analog and digital sensors and actuators that will be indispensable as

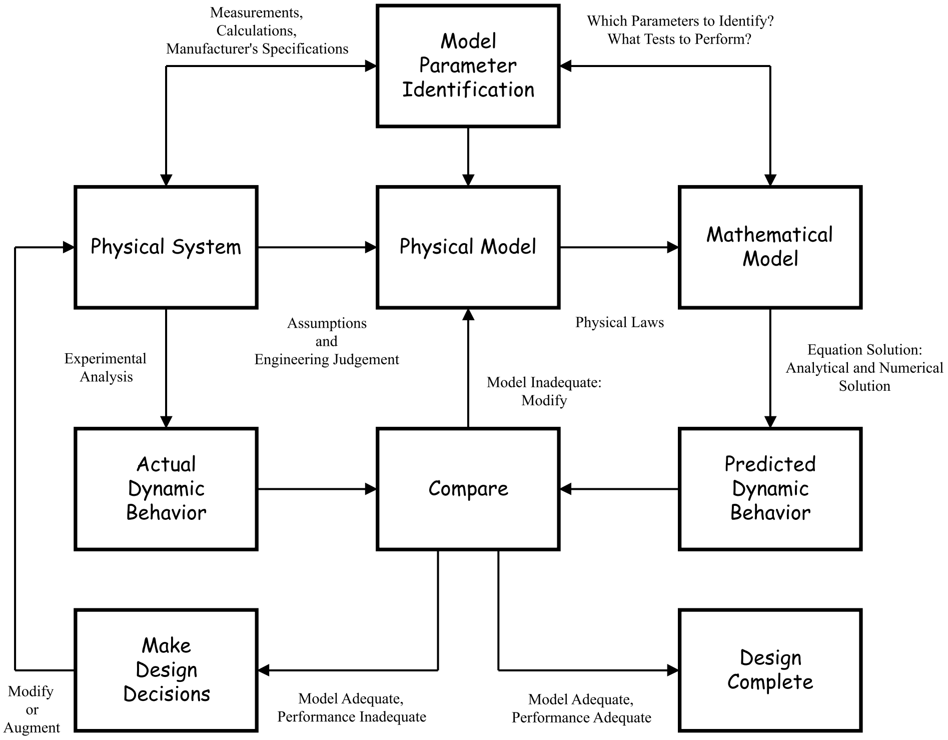

Fig. 1. Dynamic system investigation process.

mechatronic design engineers in future years. This fundamental process of dynamic system investigation shall be followed in this paper.

2. Introduction: ball-on-plate system

The ball-on-plate balancing system, due to its inherent complexity, presents a challenging design problem. In the context of such an unconventional problem, the relevance of mechatronics design methodology becomes apparent. This paper describes the design and development of a ball-on-plate balancing system that was built from an initial design concept by a team of primarily undergraduate students as part of the course *Mechatronics System Design* at Rensselaer.

Other ball-on-plate balancing systems have been designed in the past and some are also commercially available. The existing systems are, to some extent, bulky and non-portable, and prohibitively expensive for educational purposes. The objective of this design exercise, as is typical of mechatronics design, was to make the ball-on-plate balancing system '*better*, *cheaper*, *quicker*', i.e., to build a compact and affordable ball-on-plate system within a single semester. These objectives were met extremely well by the design that will be presented in this paper. The system described here is unique for its innovativeness in terms of the sensing and actuation schemes, which are the two most critical issues in this design.

The first major challenge was to sense the ball position, accurately, reliably, and in a non-cumbersome, yet inexpensive way. The various options that were considered are listed below. The relative merits and demerits are also indicated:

1. Some sort of touch-sensing scheme: not enough information available, maybe hard to implement.
2. Overhead digital camera with image-grabbing and processing software: expensive, requires the use of additional software, requires the use of a super-structure to mount the camera.
3. Resistive grid on the plate (a 2D potentiometer): limited resolution, excessive and cumbersome wiring needed.
4. Grid of infrared sensors: inexpensive, limited resolution, cumbersome, excessive wiring needed.
5. 3D-motion tracking of the ball by means of an infrared-ultrasonic transponder attached to the ball, which exchanges signals with three remotely-located towers (V-scope by Lipman Electronic Engineering): very accurate and clean measurements, requires an additional apparatus altogether, very expensive, special attachment to the ball has to be made.

Based on the above listed merits and demerits associated with each choice, it was decided to pursue the option of using a touch screen. It offered the most compact, reliable, and affordable solution. This decision was followed by extensive research pertaining to the selection and implementation of an appropriate touch sensor.

The next major challenge was to design an actuation mechanism for the plate. The plate has to rotate about its two planer body axes, to be able to balance the ball. For this design, the following options were considered:

1. Two linear actuators connected to two corners on the base of the plate that is supported by a ball-and-socket joint in the center, thus providing the two necessary degrees of motion: very expensive.
2. Mount the plate on a gimbal ring. One motor turns the gimbal providing one degree of rotation; the other motor turns the plate relative to the ring thus providing a second degree of rotation: a non-symmetric setup because one motor has to move the entire gimbal along with the plate thus experiencing a much higher load inertia as compared to the other motor.
3. Use of cable-and-pulley arrangement to turn the plate using two motors (dc or stepper): good idea, has been used earlier.
4. Use a spatial linkage mechanism to turn the plate using two motors (dc or stepper): this comprises two four-bar parallelogram linkages, each driving one axis of rotation of the plate: an innovative method never tried before, design has to be verified.

In this case, the final choice was selected for its uniqueness as a design never tried before. Fig. 2 shows an assembly view of the entire system including the spatial linkage mechanism and the touch screen mounted on the plate.

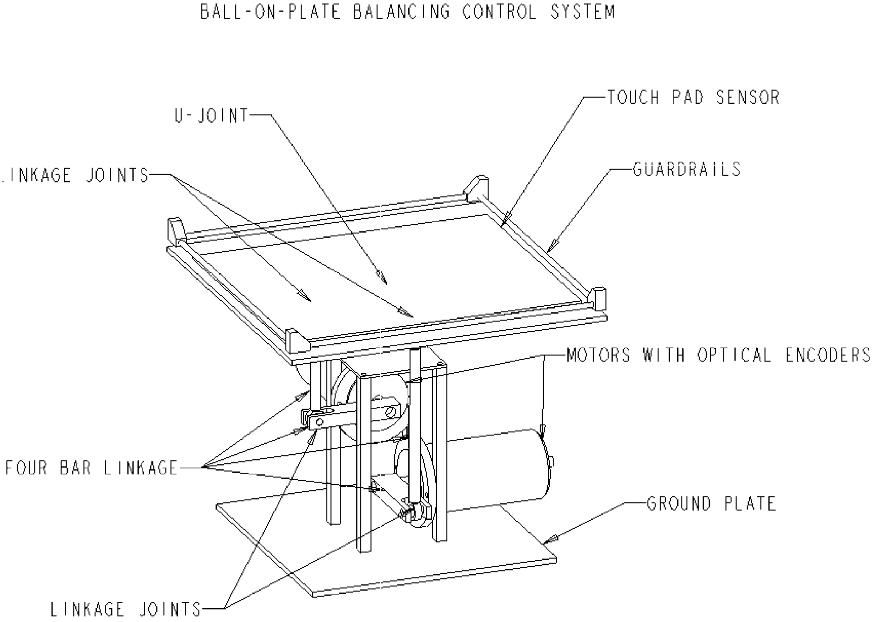

Fig. 2. Ball-on-plate system assembly.

3. Physical system description

The physical system consists of an acrylic plate, an actuation mechanism for tilting the plate about two axes, a ball-position sensor, instrumentation for signal processing, and real-time control software/hardware. The entire system is mounted on an aluminum base plate and is supported by four vertical aluminium beams. The beams provide shape and support to the system and also provide mountings for the two motors.

3.1. Actuation mechanism

Each motor drives one axis of the plate-rotation angle and is connected to the plate by a spatial linkage mechanism (Fig. 3). Referring to the schematic in Fig. 5, each side of the spatial linkage mechanism is a four-bar parallelogram linkage. This ensures that for small motions around the equilibrium, the plate angles (q_1 and q_2, defined later) are equal to the corresponding motor angles (θ_{m1} and θ_{m2}). The plate is connected to ground by means of a U-joint at O. Ball joints (at points P_1, P_2, A and B) connecting linkages and rods provide enough freedom of motion to ensure that the system does not bind. The motor angles are measured by high-resolution optical encoders mounted on the motor shafts. A dual-axis inclinometer is mounted on the plate to measure the plate angles directly. As shall be shown later, for small motions, the motor angles correspond to the plate angles due to the kinematic constraints

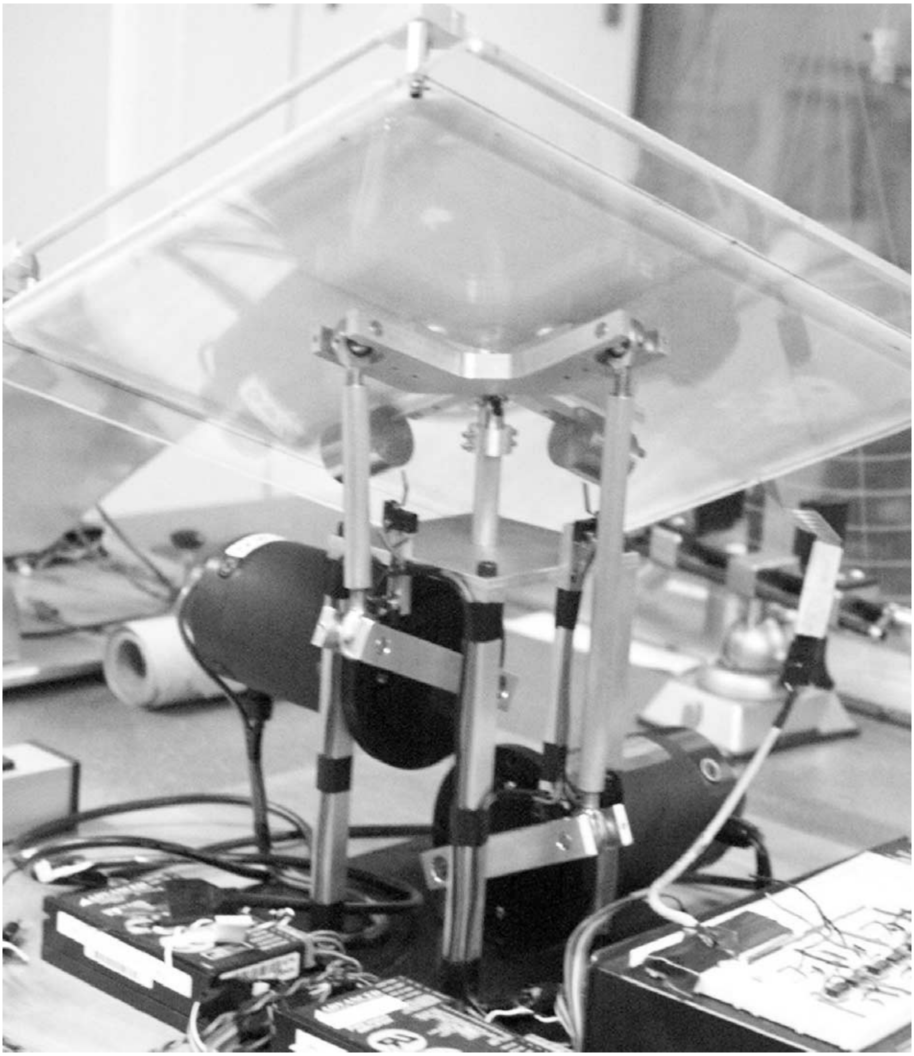

Fig. 3. The spatial linkage mechanism used for actuating the plate.

imposed by the parallelogram linkages. The motors used for driving the linkage are simple brushed dc motors.

3.2. PWM servo-amplifiers

The motors are operated in current mode for ease of modeling and controls. A pulse-width-modulated servo-amplifier operating in voltage-to-current amplification mode is employed for this purpose. The amplifiers are powered by a 24 V dc power supply.

3.3. Ball-position sensor

A resistive touch-sensitive glass screen (TouchTek from MicroTouch) that is actually meant to be a computer touchscreen was used for sensing the ball position. It provides an extremely reliable (less than 1% error), accurate (1024 × 1024 points across the screen), and economical solution to the ball-position-sensing problem. The screen consists of three layers: a glass sheet, a conductive coating on the glass sheet, and a hard-coated conductive top-sheet. An external dc voltage is applied to the four corners of the glass layer. Electrodes spread out the voltage on the glass layer creating a uniform voltage field. When the top layer is pressed by the weight of the ball, the top sheet gets compressed into contact with the conductive coating on the glass layer. As a result, current is drawn from each side of the glass layer in proportion to the distance of the touch from the edge. This generates a set of four voltages at the corners of the glass sheet. These four voltages are filtered and subsequently used for computing the ball position coordinates (x_b and y_b) using simple linear relationships. The response time of this sensor is 8–15 ms which is fast enough for this application. The ball rolls on this touch screen, which in turn is mounted on the acrylic plate.

3.4. Real-time controls implementation

A Matlab/Simulink-based real-time control prototyping application dSpace is used for implementing the controller design for this system.

4. Physical system modeling and assumptions

The following assumptions are used in the modeling the above-described physical system:

1. It is assumed that the sliding friction between the ball and plate is high enough to prevent the ball from slipping on the plate. This limits the degrees of freedom of the system and also makes the equations of motion simpler.
2. The rotation of the ball about its vertical axis is assumed to be negligible.
3. Rolling friction between the ball and the plate is neglected.
4. It is assumed that there will be small motion of the plate about the equilibrium configuration. This ensures that the plate angles will be approximately equal to motor angles.
5. The plate is assumed to have mass-symmetry about its x–z and y–z planes. This ensures that there are no non-diagonal terms in the inertia matrix for the plate.

A physical model of the ball-on-plate system is provided in Fig. 4, where x–y–z is the ground frame. The plate has two degrees of freedom and its orientation is defined by two angles (q_1 and q_2) that constitute a body (1–2) rotation. Frame x''–y''–z'' is a plate fixed reference frame, while x'–y'–z' is an intermediate frame. All angles are defined to be positive in the CCW sense.

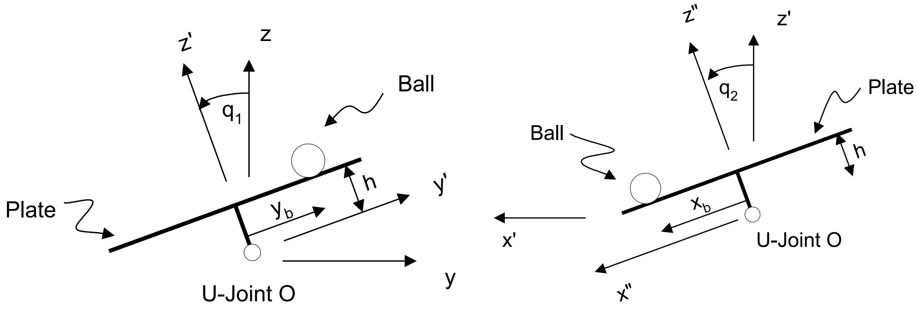

Fig. 4. Physical model of the ball-on-plate system.

5. Mathematical model

5.1. Kinematic analysis

A linkage diagram of the spatial linkage is shown in Fig. 5. The L-shaped link is rigidly attached to the base of the plate and is connected to the ground by means of a U-joint at *O*. The two motors are connected to links at simple pin joints. The remaining joints are ball-and-socket joints.

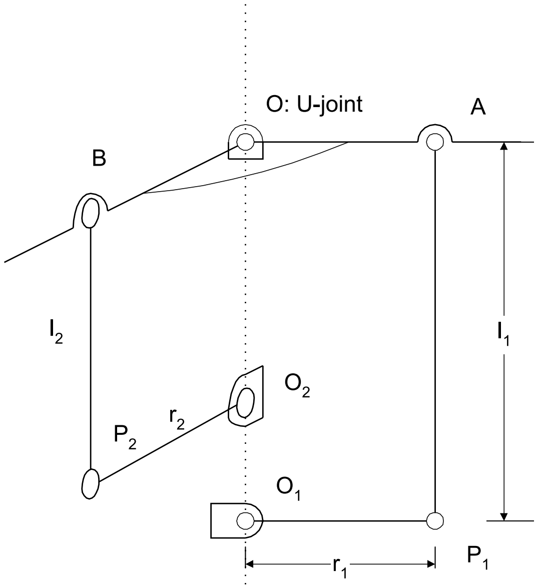

Fig. 5. Plate actuation spatial linkage mechanism.

5.1.1. Degree of freedom calculation

Number of rigid bodies $(n) = 5$;
Number of pin joints $(p) = 2$;
Number U-joints $(u) = 1$;
Number of ball-and-socket joints = 4;
Number of redundant degrees of freedom $(r) = 2$

(Note that the rotations of the two vertical links about their respective axes constitute two redundant degrees of freedom, since they do not influence the system state in any way)

Hence the overall degrees of freedom of the system

$$= 6(n) - 5(p) - 4(u) - 3(b) - r = 2.$$

Thus the plate and spatial linkage mechanism has two degrees of freedom, which is as expected. This is also equal to the inputs to the system, the two motor angles.

5.1.2. Relationship between motor angles and plate angles

From the above discussion, it is evident that out of the four variables (θ_{m1}, θ_{m2}, q_1, and q_2), only two are independent since the mechanism has two degrees of freedom. Thus, there exist the following two kinematic constraint equations that relate the motor angles to the plate angles

$$\begin{aligned}&(r_1\cos q_1 - r_1\cos\theta_{\mathrm{m1}})^2 + (r_1\sin q_1 - r_1\sin\theta_{\mathrm{m1}} + l_1)^2 = l_1^2,\\&(r_2\sin\theta_{\mathrm{m2}} - r_2\sin q_2\cos q_1 + l_2)^2 + (r_2\cos q_2 - r_2\cos\theta_{\mathrm{m2}})^2\\&\quad + (r_2\sin q_2\sin q_1) = l_2^2.\end{aligned} \tag{1}$$

It is noticed that, in general the plate angle q_2 is related to the motor angles θ_{m1} and θ_{m2} by the highly non-linear equations presented above. Nevertheless, for small motions about the equilibrium, it can easily be shown that the above expressions reduce to the following linear relationships:

$$\begin{aligned}&q_1 = \theta_{\mathrm{m1}} \quad \text{(Always true)},\\&q_2 \cong \theta_{\mathrm{m2}} \quad \text{(True for small motion)}.\end{aligned} \tag{2}$$

This is the ‘small angle’ assumption that was listed earlier in this paper. The validity of this assumption is also verified experimentally. It is found that for the relevant range of operation, the correspondence between the encoder reading (motor angles) and the inclinometer reading (plate angle) is very satisfactory.

5.2. Dynamic system analysis

The equations of motion for this system can be derived using either using Newton’s laws or Lagrange’s equations. For this case, both the methods were used to verify the final results. The complete non-linear and coupled set of equations is given by:

$$\begin{aligned}
x\text{-direction}: \; & m_{\mathrm{b}} g r_{\mathrm{b}} \sin q_2 \cos q_1 - m_{\mathrm{b}} r_{\mathrm{b}}[(h + r_{\mathrm{b}})\ddot{q}_2 - y_{\mathrm{b}}\ddot{q}_1 \sin q_2 - x_b \dot{q}_2^2 \\
& - x_{\mathrm{b}}\dot{q}_1^2 \sin^2 q_2 + (h + r_{\mathrm{b}})\dot{q}_1^2 \sin q_2 \cos q_2 - 2\dot{y}_{\mathrm{b}}\dot{q}_1 \sin q_2 + \ddot{x}_{\mathrm{b}}] \\
& - I_{\mathrm{b}}((\ddot{x}_{\mathrm{b}}/r_{\mathrm{b}}) + \ddot{\mathrm{q}}_2) = 0,
\end{aligned} \tag{3}$$

$$\begin{aligned}
y\text{-direction}: \; & (m_{\mathrm{b}} g r_{\mathrm{b}} \sin q_1) + m_{\mathrm{b}} r_{\mathrm{b}}[x_{\mathrm{b}}(\ddot{q}_1 \sin q_2 + \dot{q}_2\dot{q}_1 \cos q_2) \\
& - (h + r_{\mathrm{b}})(\ddot{q}_1 \cos q_2 - \dot{q}_2\dot{q}_1 \sin q_2) + \dot{q}_2\dot{q}_1(h_{\mathrm{b}} + r_{\mathrm{b}}) \sin q_2 \\
& - y_{\mathrm{b}}\dot{q}_1^2 + x_{\mathrm{b}}\dot{q}_2\dot{q}_1 \cos q_2 + 2\dot{x}_{\mathrm{b}}\dot{q}_1 \sin q_2 + \ddot{y}_{\mathrm{b}}] + I_{\mathrm{b}}((\ddot{y}_{\mathrm{b}}/r_{\mathrm{b}}) \\
& - \ddot{q}_1 \cos q_2 + \dot{q}_2\dot{q}_1 \times \sin q_2) = 0.
\end{aligned} \tag{4}$$

The equations of motion in this non-linear form are of little use in terms of designing a controller based on linear controls theory. Hence, these equations are linearized about the operating point, which is the equilibrium configuration ($q_1 = q_2 = x_{\mathrm{b}} = y_{\mathrm{b}} = 0$). Interestingly, linearization also decouples the two modes of motion

$$\begin{aligned}
& \frac{7}{5}\ddot{x}_{\mathrm{b}} + \left(\frac{7}{5}r_{\mathrm{b}} + h\right)\ddot{q}_2 = g q_2, \\
& \frac{7}{5}\ddot{y}_{\mathrm{b}} - \left(\frac{7}{5}r_{\mathrm{b}} + h\right)\ddot{q}_1 = -g q_1.
\end{aligned} \tag{5}$$

These lead to the following transfer functions for a particular set of parameter values:

$$\begin{aligned}
& \frac{x_{\mathrm{b}}}{q_2} = \frac{-0.035 s^2 + 7}{s^2}, \\
& \frac{y_b}{q_1} = \frac{0.035 s^2 - 7}{s^2}.
\end{aligned} \tag{6}$$

5.3. Control system design

From the linearized set of equations, it is seen that x_{b} is dependent on q_2 only, while y_{b} is dependent on q_1 only. Thus the system can be treated as two different systems operating simultaneously. Hence, similar but independent controllers can be used for controlling each coordinate of the ball motion. The design of only one controller is discussed here. The other one is exactly the same.

Based on the linear model, a preliminary controller is designed with the scheme of a 'loop within a loop'. The first step involves the design of an inner loop where the encoder feedback is sent to the dc motors to achieve a servo position control. A simple PID controller is adequate to obtain a very high response speed which is ideally desired for the inner loop.

The inner loop is then placed in an outer loop that controls the ball position. The next step in the control system design is to obtain a controller for the outer loop, based on the transfer function between ball position and the corresponding plate angle (x_b vs. q_2). Using root-locus design techniques, an appropriate lead controller is easily designed to achieve this objective.

The overall control scheme can be explained as follows. While the controller in the outer loop computes the angle by which the plate should move to balance the ball, the inner loop controller actually moves the plate by that angle. Ideally, the inner

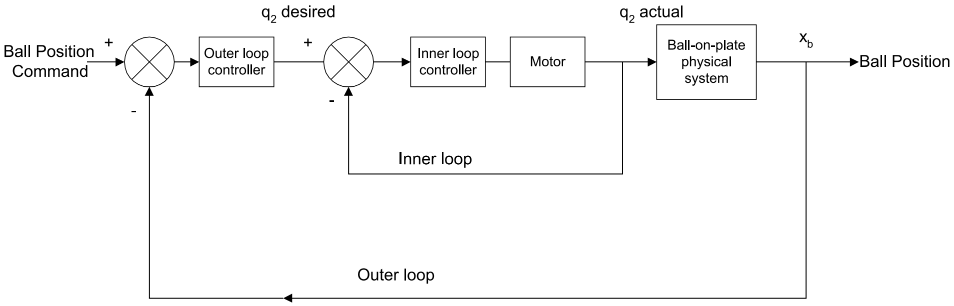

Fig. 6. Control scheme.

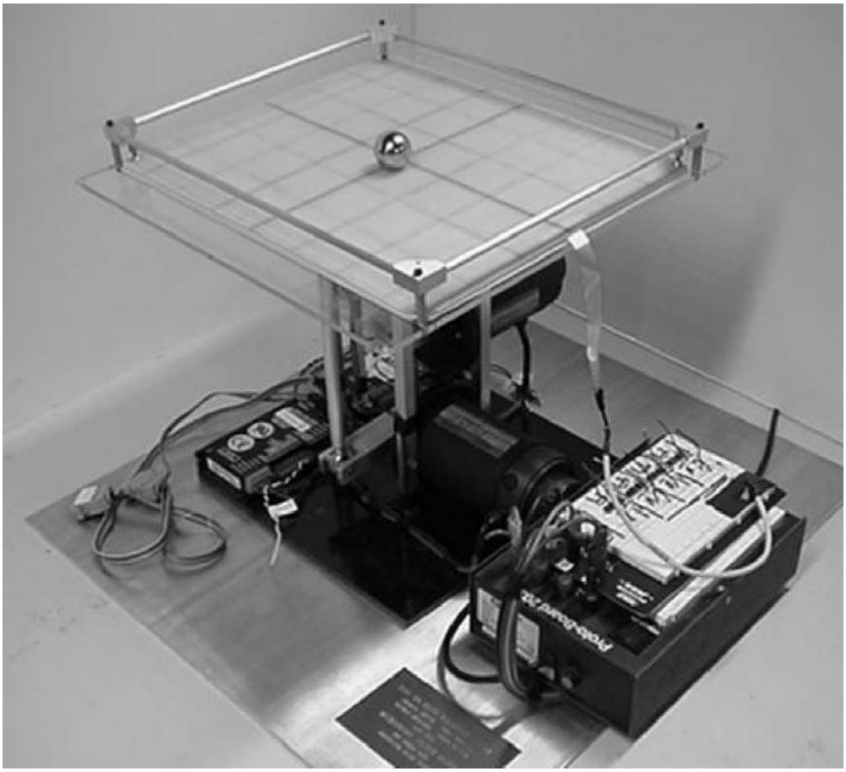

Fig. 7. The ball-on-plate system in operation.

loop should do this instantaneously, which is not possible in reality. Nevertheless, it is desirable to keep the speed of the inner loop much higher than that of the outer loop. This simple scheme is extremely effective in achieving the desired objective of balancing the ball (Fig. 6). Fig. 7 is a picture of the ball-on-plate balancing system in action.

6. Results and future work

Using the real-time control prototyping tool dSpace, the control scheme discussed above was actually implemented on the ball-on-plate system. Despite being based on a linearized model, the controllers performed extremely well with the non-linear system. When the system is in operation, the ball can be commanded to stay balanced at any point on the plate. It can also be directed to move from one point to another, and stay there. In fact, using this control scheme the ball can even trace any desired path on the plate, for example a circle or a figure eight. (See the website http://mechatronics.meche.rpi.edu for a movie.)

The system once built and tested can be further used as an excellent test-bed for testing various other control schemes. An optimal controller using full-state feedback can be designed to achieve yet better performance. Although controllers based on the linear model perform extremely well, it will be interesting to apply the principles of non-linear controls and seek any further improvements in the system performance.