# Large Stroke Electrostatic Comb-Drive Actuators Enabled by a Novel Flexure Mechanism

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Abstract—This paper presents in-plane electrostatic comb-drive actuators with stroke as large as 245  $\mu$ m that is achieved by employing a novel clamped paired double parallelogram (C-DP-DP) flexure mechanism. The C-DP-DP flexure mechanism design offers high bearing direction stiffness  $K_x$  while maintaining low motion direction stiffness  $K_y$  over a large range of motion direction displacement. The resulting high  $(K_x/K_y)$  ratio mitigates the onset of sideways snap-in instability, thereby offering significantly greater actuation stroke compared with existing designs. Further improvement is achieved by reinforcing the individual beams in this flexure mechanism. While the traditional paired double parallelogram (DP-DP) flexure design with comb gap  $G = 3 \ \mu m$ and flexure beam length  $L_1 = 1$  mm results in a 50- $\mu$ m stroke before snap-in, the reinforced C-DP-DP design with the same comb gap and flexure beam length achieves a stroke of 141  $\mu$ m. Furthermore, this C-DP-DP flexure design provides a  $215 - \mu m$ stroke with  $G = 4 \ \mu m$  and a 245- $\mu m$  stroke with  $G = 6 \ \mu m$ . The presented work includes closed-form stiffness expressions for the reinforced C-DP-DP flexure, a design procedure for selecting dimensions of the overall comb-drive actuator, microfabrication of some representative actuators, and experimental measurements [2012-0067] demonstrating the large stroke.

*Index Terms*—Comb drive, electrostatic actuator, flexure mechanism, large stroke.

## I. INTRODUCTION AND BACKGROUND

**E** LECTROSTATIC microelectromechanical systems (MEMS) comb-drive actuators have been used in various applications such as resonators [1], filters [2], microgrippers [3], [4], and micro/nano positioning [5], [6]. A linear in-plane electrostatic comb-drive actuator comprises two electrically isolated conductive combs with N fingers each, schematically shown in Fig. 1. While the static comb is fixed with respect to ground, the moving comb is guided via a flexure mechanism so that it can displace primarily in the **Y**-direction with respect to the static comb. These static and moving comb fingers (length

Manuscript received March 15, 2012; revised October 19, 2012; accepted October 24, 2012. Date of publication December 12, 2012; date of current version March 29, 2013. This work was supported in part by the National Science Foundation under Grant CMMI-0846738. The work of S. Sood was supported by the National Institute of Science and Technology under a Measurement Science and Engineering Graduate Fellowship. The experimental portion of this work was performed at the Lurie Nanofabrication Facility, a member of the National Nanotechnology Infrastructure Network, which is supported in part by the National Science Foundation. Subject Editor L. Lin.

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Digital Object Identifier 10.1109/JMEMS.2012.2227458



Fig. 1. Schematic of an electrostatic comb drive with the springs representing the flexure bearing.

 $L_f$ , in-plane thickness  $T_f$ , and out-of-plane thickness  $H_f$ ) have an interdigitation gap of G and an initial engagement of  $Y_o$ . The flexure mechanism is designed to provide linear guided motion and relatively small stiffness  $K_y$  in the **Y**-direction (or motion direction), along with minimal error motions  $E_x$  and relatively high stiffness  $K_x$  in the X-direction (or bearing direction). In an ideal scenario,  $K_y$  and  $E_x$  would approach zero while  $K_x$ would approach infinity. However, in practice, this is never the case given the performance tradeoffs between motion range, stiffness, and error motions that exist in flexure mechanisms [7], and manufacturing imperfections that are inherent to microfabrication processes [8], [9]. High stiffness  $K_{\theta}$  and low error motion  $E_{\theta}$  are also desirable in the in-plane yaw rotation [10]. Since  $K_{\theta}$  can be independently made large and  $E_{\theta}$  is inherently zero for the flexure designs considered here, in-plane rotation is initially ignored. The three out-of-plane directions are also bearing directions, but by appropriate choice of dimensions, become noncritical in the overall performance of the in-plane comb-drive actuator [11].

When a voltage difference V is applied between the two combs, they experience an electrostatic attractive force, which displaces the moving comb by Y along the motion direction

$$K_y \cdot Y = \frac{\varepsilon N H_f G}{G^2 - X^2} V^2. \tag{1}$$

Here,  $\varepsilon$  is the dielectric constant of air. Nonzero bearing direction displacement X can arise due to flexure error motion, fabrication misalignment, electrostatic forces, or a possible disturbance force in the X-direction. While displacement Y is dictated by the comb geometry, motion direction flexure stiffness, and actuation voltage, its maximum stroke is limited by the snap-in phenomenon, which corresponds to sideways

instability of the moving comb [12], [13]. For any Y displacement, the electrostatic force due to the actuation voltage V produces a destabilizing or negative spring effect and the flexure mechanism offers a stabilizing or positive spring effect in the X-direction. The former increases with increasing stroke, whereas the latter generally reduces. At the Y displacement when the former stiffness exceeds the latter, the moving comb snaps sideways into the static comb. This condition may be mathematically expressed as [12], [14]

$$\left(\frac{K_x}{K_y}\right) \le \frac{2Y(Y+Y_0)}{G^2} \frac{\left(1 + \frac{3X_c^2}{G^2}\right)}{\left(1 - \frac{X_c^2}{G^2}\right)^2}, \text{ where } E_x = \frac{4X_c^3}{G^2 + 3X_c^2}.$$
 (2)

This snap-in condition assumes that the comb fingers are perfectly rigid and all compliance comes from the flexure mechanism. Moreover, the yaw rotational stiffness  $K_{\theta}$  of the flexure mechanism is assumed large enough to be ignored. The right-hand side represents a "critical stiffness ratio" needed to avoid snap-in and clearly increases with displacement Y and error motion  $E_x$ .  $E_x$  includes any motion or misalignment of the moving comb in the X-direction with respect to its nominal zero position due to flexure mechanics or fabrication imperfections, in the absence of electrostatic force. Here,  $X_c$  is the actual X-direction displacement at the onset of an electrostatic snap-in when an error or misalignment  $E_x$  is present. Clearly, to delay snap-in and maximize the actuator stroke, the flexure mechanism should provide a high  $(K_x/K_y)$  ratio and low  $E_x$ that are maintained over a large Y displacement range. Given the uncertainty in  $E_x$  from fabrication imperfections, its effect may be incorporated via a positive stability margin S. Stable operation is given by

$$\left(\frac{K_x}{K_y}\right) \ge \frac{2Y(Y+Y_0)}{G^2}(1+S),$$
  
where  $S = \left(1 + \frac{3X_c^2}{G^2}\right) \left/ \left(1 - \frac{X_c^2}{G^2}\right)^2 - 1.$  (3)

From (2), it can be shown that for an error motion that is 16% of the comb gap (e.g.,  $E_x = 0.64 \ \mu \text{m}$  and  $G = 4 \ \mu \text{m}$ ), a stability margin of 1 is required.

Large actuation stroke (> 100  $\mu$ m) with limited footprint and actuation effort is desirable in a wide range of MEMS applications, including optical switches [12], data-storage systems [15], high-resolution microprinting [16], [17], endoscopic microscopy [18], neural microelectrodes [19], and scanning probe microscopy [5]. The broad goal of this paper is to investigate and overcome the fundamental limits in flexure mechanism design to maximize the stroke of MEMS comb-drive actuators while minimizing the device footprint and actuation voltage, for any given application. To meet this goal, we introduce a novel flexure mechanism, i.e., the clamped paired double parallelogram (C-DP-DP) flexure, shown in Fig. 2.

Section II provides a review of the prior art on flexure mechanism designs that have been traditionally used with comb-drive actuators and their limitations. The proposed C-DP-DP flexure mechanism, which offers a high  $(K_x/K_y)$  ratio over a large Y displacement range, is presented in Section III, along with closed-form analytical expressions for its stiffness. Section IV



Fig. 2. Clamped paired double parallelogram (C-DP-DP) flexure.



Fig. 3. Parallelogram (P) flexure.

outlines a step-by-step recipe to optimally select the dimensions of this flexure mechanism and the comb drive to design an actuator with large stroke while minimizing actuation voltage and device footprint. Microfabrication of some representative actuators and experimental results are elaborated in Section V. Using the C-DP-DP flexure, a maximum actuation stroke of 245  $\mu$ m with a 1-mm flexure beam length and a 6- $\mu$ m comb gap is demonstrated.

## II. PRIOR ART

Some of the earliest flexure designs used in comb-drive applications include the crab-leg flexure [13] and the folded-beam flexure [20], but both have limitations. While the former exhibits a  $K_y$  stiffness value that considerably grows with Y displacement, the latter provides a suboptimal  $K_x$  stiffness value that further deteriorates with increasing Y displacement [11].

The parallelogram (P) flexure, as shown in Fig. 3, is a common single-axis flexure mechanism and serves as an

important building block in other flexure mechanisms that have been widely used in comb-drive actuators in the past and in the new flexure mechanism presented in this paper. Therefore, it is discussed in further detail here. In the P flexure shown in Fig. 3, a general shape is assumed for each constituent beam, with two equal end segments having uniform thickness  $T_1$  and length  $a_0L_1$  and a rigid middle section of length  $(1 - 2a_0)L_1$ . Geometric parameter  $a_0$  quantifies the degree of distributed compliance:  $a_0 = 1/2$  represents a uniform thickness beam with highly distributed compliance, whereas smaller values of  $a_0$  correspond to increasingly lumped compliance. In the subsequent discussion,  $a_0$  serves as a geometric shape optimization parameter.

Closed-form nonlinear stiffness and error motion relations for this flexure have been previously derived [7] and are summarized here

$$K_y = \frac{2EI_1}{L_1^3} k_{11}^{(0)} \tag{4}$$

$$K_x = \frac{2EI_1}{L_1^3} \frac{k_{33}}{\left(1 + k_{33}g_{11}^{(1)} \left(\frac{Y}{L_1}\right)^2\right)}$$
(5)

$$K_{\theta} = \frac{2EI_1}{L_1^3} \cdot W^2 \cdot \frac{k_{33}}{\left(1 + k_{33}g_{11}^{(1)}\left(\frac{Y}{L_1}\right)^2\right)}$$
(6)

$$E_x = -\frac{k_{11}^{(1)}}{2} \frac{Y^2}{L_1} \tag{7}$$

$$E_{\theta} = \frac{k_{11}^{(0)}}{2W^2} \left(\frac{Y}{L_1}\right) \left(\frac{1}{k_{33}} + g_{11}^{(1)} \left(\frac{Y}{L_1}\right)^2\right).$$
(8)

Here, E is the Young's modulus of the material, and  $I_1$  is the second moment of area  $(=H_1T_1^3/12)$ . Nondimensional terms  $k_{11}^{(0)}$ ,  $k_{11}^{(1)}$ ,  $g_{11}^{(1)}$ , and  $k_{33}$  are all functions of the beam shape  $(a_o \text{ and } T_1)$  and are referred to as beam characteristic coefficients [21]. These are graphically illustrated in Fig. 3 and mathematically expressed as follows:

$$k_{11}^{(0)} = \frac{6}{(3 - 6a_o + 4a_o^2) a_o} \\ k_{11}^{(1)} = \frac{3(15 - 50a_o + 60a_o^2 - 24a_o^3)}{5(3 - 6a_o + 4a_o^2)^2} \\ g_{11}^{(1)} = \frac{2a_o^3(105 - 630a_o + 1440a_o^2 - 1480a_o^3 + 576a_o^4)}{175(3 - 6a_o + 4a_o^2)^3} \\ k_{33} = \frac{6}{a_0(T_1/L_1)^2}.$$
(9)

Per (4), the P flexure provides low  $K_y$  stiffness that remains constant with Y and depends directly on coefficient  $k_{11}^{(0)}$ .  $k_{11}^{(0)}$ , which corresponds to the normalized elastic stiffness of an individual beam in the Y-direction, is insensitive to values of  $a_0$  around 0.5 but significantly grows as  $a_o$  decreases [see Fig. 4(b)]. Per (5), this flexure also provides high  $K_x$  stiffness at Y = 0, which depends on coefficient  $k_{33}$ .  $k_{33}$  is the nominal elastic stiffness of an individual beam in the X-direction normalized with respect to its bending stiffness and also increases with reducing  $a_0$  [see Fig. 4(a)].



Fig. 4. Beam characteristic coefficients. For the  $k_{33}$  plot,  $(T_1/L_1)$  is assumed to be 0.003.

Furthermore, the stiffness  $K_x$  drops with increasing Y displacement due to the  $k_{33}g_{11}^{(1)}$  term in the denominator of (5). Specifically, coefficient  $g_{11}^{(1)}$  captures the elastokinematic effect in an individual beam that contributes additional **X**-direction compliance in the presence of a Y displacement, and approaches zero with reducing  $a_0$  [see Fig. 4(b)]. Since the product  $k_{33}g_{11}^{(1)}$  also approaches zero with reducing  $a_0$  (see Fig. 4), the drop in  $K_x$  stiffness can be reduced by decreasing  $a_0$ .

As given by (6), the rotational stiffness  $K_{\theta}$  of this flexure mechanism follows a trend similar to  $K_x$ . However, the nominal value of this stiffness can be independently increased to be large enough by increasing the dimension W.

Thus, as evident in Fig. 6, the P flexure provides a favorable  $K_x/K_y$  stiffness ratio that can be further optimized by reducing  $a_o$ . However, the overwhelming disadvantage of this flexure mechanism is its large error motion  $E_x$  that arises due to the kinematics of beam arc-length conservation, which is captured via coefficient  $k_{11}^{(1)}$ . Since this kinematic coefficient is fundamental to the geometry of the beam, it always remains between 1 and 1.2, irrespective of the beam shape  $a_0$ , as evident in Fig. 4(a). The quadratic relation given by (7) shows that the motion stage in a P flexure follows a parabolic trajectory instead of a perfectly straight line. For typical beam dimensions  $(L_1 = 1 \text{ mm and } a_0 = 0.5)$  and displacement  $Y = 0.1L_1$ , the  $E_x$  error motion is 6  $\mu$ m, which is significant and would cause very early snap-in, as predicted by (2). One effective method to alleviate this issue without eliminating  $E_x$  is to systematically prebend the beams of the P flexure, introduce an initial offset in the gap between the fixed and moving combs, and incline the comb fingers with respect to the Y-axis [22]. Grade *et al.* [23] implemented this clever idea to demonstrate a 175- $\mu$ m comb-drive actuator stroke using  $L_1 = 2$  mm and asymmetric comb gaps of 6 and 9  $\mu$ m. While snap-in is thus mitigated, the trajectory of the motion stage is still parabolic, which can be undesirable in certain applications.

In order to eliminate the above  $E_x$  error motion, one might employ two P flexures in a symmetric parallel configuration (P-P) [24]. However, this results in an overconstrained geometry that produces a quadratic rise in the motion direction stiffness with increasing Y displacement and therefore a significantly restricted stroke. A more appropriate approach is to use two oppositely oriented P flexures in series. The resulting double parallelogram (DP) flexure employs the principle of geometric reversal to exactly cancel out the kinematic error motion of one P with that of the other P, in the absence of X-direction force, to produce  $E_x = 0$ . Next, employing two DP flexures in parallel, as shown in Fig. 7, produces a symmetrical geometry that ensures  $E_{\theta} = 0$  and reduced sensitivity to fabrication imperfections. In fact, the earliest reported combdrive actuators employed the paired double parallelogram DP-DP flexure [1]. Subsequently, Legtenberg et al. [13] extensively studied this flexure for comb-drive actuation and reported a 39.9- $\mu$ m stroke with  $L_1 = 500 \ \mu$ m and  $G = 2.2 \ \mu$ m.

Closed-form nonlinear stiffness relations for the DP-DP flexure have been also previously derived [7] and are summarized here

$$K_y = \frac{2EI_1}{L_1^3} k_{11}^{(0)} \tag{10}$$

$$K_x = \frac{2EI_1}{L_1^3} \frac{k_{33}}{\left(1 + k_{33} \left(g_{11}^{(1)} + \frac{\left(k_{11}^{(1)}\right)^2}{k_{11}^{(0)}}\right) \left(\frac{Y}{2L_1}\right)^2\right)}$$
(11)

$$K_{\theta} = \frac{EI_1}{L_1^3} \cdot \frac{4W_1^2 W_2^2}{(W_1^2 + W_2^2)} \cdot \frac{k_{33}}{\left(1 + k_{33} g_{11}^{(1)} \left(\frac{Y}{2L_1}\right)^2\right)}.$$
 (12)

The motion direction stiffness  $K_y$  of the DP-DP flexure is low and remains largely invariant with Y displacement, i.e., same as that of the P flexure. However, while its nominal  $K_x$ stiffness at Y = 0 is the same as that of the P flexure, the drop in  $K_x$  with increasing Y displacement is far more precipitous. The reason being that, in addition to the  $k_{33}g_{11}^{(1)}$  term in the denominator of (11), there is now a new term, i.e.,  $(k_{11}^{(1)})^2 k_{33}/k_{11}^{(0)}$ , which is at least two orders of magnitude greater than the former. Since this term is dependent on kinematic coefficient  $k_{11}^{(1)}$ , it is largely insensitive to variations in  $a_0$  and never approaches zero (see Fig. 5). Therefore, while the nominal  $K_x/K_y$  ratio at Y = 0 for a DP-DP flexure can be increased by employing a lower value of  $a_o$ , the steep decline in this stiffness ratio remains unaffected by beam shape variation (see Fig. 6). In this figure, a representative critical stiffness ratio curve for a typical comb-drive actuator demonstrates how the narrow profile of the



Fig. 5. Elastokinematic and kinematic contributions to the bearing direction compliance of the P and DP-DP flexures.



Fig. 6.  $(K_x/K_y)$  stiffness ratio provided by the P, DP-DP, and C-DP-DP flexures for different values of beam reinforcement  $a_o$ . A representative critical stiffness curve is included to demonstrate the effect of stiffness ratio on comb-drive actuator snap-in.



Fig. 7. Paired double parallelogram (DP-DP) flexure.

 $K_x/K_y$  ratio versus Y displacement produces an early snap-in and limited stroke when the DP-DP flexure is used.

It is noteworthy that the in-plane rotational stiffness  $K_{\theta}$  in this case, given by (12), follows a trend very similar to the P



Fig. 8. Degradation in the stability margin with increasing beam prebend in DP-DP flexures.

flexure. As a result, the drop in  $K_{\theta}$  stiffness from its nominal value with increasing Y displacement is far more gradual. In addition, this stiffness can be independently made large by appropriate choice of the dimensions  $W_1$  and  $W_2$ .

Recognizing the limitation of this narrow  $K_x/K_y$  stiffness ratio profile and the fact that its maximum value occurs at Y = 0 where the required critical stiffness ratio is minimum, DP and DP-DP flexures with prebent [25] or pretilted [26] beams have been used to improve the comb-drive actuator stroke. Appropriately prebent beams shift the Y displacement value at which  $K_x$  is maximum, without abating the drop in  $K_x$  or affecting  $K_y$  [12]. Thus, by providing a greater  $K_x/K_y$ stiffness ratio at larger displacements, where it is most needed, prebending provides improvement in the actuator stroke. With 500- $\mu$ m beam length, 30- $\mu$ m pretilt, and 2- $\mu$ m comb gap, Zhou and Dowd [26] reported a 61- $\mu$ m stroke, which was two times higher compared with a similar DP-DP flexure without any pretilt. Grade et al. [12] also reported strokes as high as 150  $\mu$ m with a 1.1-mm beam length, approximately 100- $\mu$ m prebend, and 8.5- $\mu$ m comb gap. However, these improvements in stroke come at the expense of robustness [14], as illustrated in Fig. 8.

For a given set of DP-DP dimensions, greater prebending causes the stability margin S to fall to levels below 1 over the expected actuator stroke. This makes the design vulnerable to premature snap-in, given the presence of finite fabrication imperfections. It can be shown that, to keep S greater than 1 over the actuation stroke, the maximum prebend in a DP or DP-DP flexure should be less than

$$Y_{P-B-\max} \approx \sqrt{\frac{4GL_1}{k_{11}^{(1)}}}.$$
 (13)

Here,  $Y_{P-B-\max}$  represents the overall shift in the  $K_y$  stiffness peak along the Y displacement axis due to prebending of beams. For the typical values of  $L_1 = 1 \text{ mm}$ ,  $G = 3 \mu \text{m}$ , and  $a_0 = 0.5$ , the maximum allowable prebend is 100  $\mu$ m, as shown in Fig. 8.

Thus, in the flexure designs considered in the literature so far, there exist clear performance tradeoffs between stiffness, error motions, and robustness, which affect their ability to provide very large strokes in comb-drive actuators. The desirable goal in a flexure mechanism design is to ensure inherently low error motions, achieve a high nominal value of the  $K_x/K_y$  ratio, and prevent the steep decline of this stiffness ratio with increasing Y displacement. As shown in Fig. 5, these stiffness goals are accomplished via the new C-DP-DP flexure design presented in this paper.

However, before describing the C-DP-DP flexure design in the next section, it is important to first physically understand the source of the steep decline in  $K_x$  seen in the DP and DP-DP mechanisms, as well as their prebent versions. As noted earlier, the kinematic effect associated with arc-length conservation is fundamental to a beam flexure and does not go away simply by beam shape optimization. It also produces a loadstiffening effect in each beam flexure that linearly increases the Y-direction stiffness in the presence of an X-direction force. The reason these kinematic and load-stiffening effects play an important role in the DP-DP flexure's  $K_x$  stiffness is because of the mechanism's topology. In this design, the secondary stages of both DPs are inadequately constrained in the Y-direction. As a consequence, when the Y displacement of the motion stage is held fixed and a small bearing direction force  $F_x$  is applied, the two secondary stages move opposite to each other in the motion direction from their nominal displacement of Y/2. This is because the Y-direction stiffness of each of the P flexures within the two DPs changes in the presence of  $F_x$  due to the above load-stiffening effect. In the presence of this "extra" Y-direction displacement of the two secondary stages, the kinematic errors of the individual P flexures in the X-direction no longer cancel out perfectly, thereby producing an "extra" X displacement at the motion stage and therefore lower  $K_x$ .

Thus, the above-described steep decline in  $K_x$  can be restricted by appropriately constraining the Y-direction displacement of both secondary stages such that they always remain at their nominal value of Y/2, i.e., at half that of the motion stage. This was recognized several decades ago in the context of precision motion guidance instruments, and an external-leverbased solution was proposed [27]. A lever arm was employed to kinematically enforce a 1:2 ratio between the Y-direction displacements of the secondary stage and the motion stage in a DP flexure. A microfabricated variation of this design has been used by Brouwer et al. [28] to successfully restrict the drop in  $K_x$  stiffness and improve the comb-drive actuation stroke. They reported a 100- $\mu$ m stroke with a 1-mm beam length and a 4- $\mu$ m comb gap. However, certain challenges remain with this design. The use of an external lever increases motion direction stiffness  $K_y$ , which is detrimental to the  $K_x/K_y$  ratio and increases actuation effort. In addition, this design limits access to the motion stage and increases the overall device footprint, as compared with the basic DP flexure.

Apart from the flexure mechanism design, several other techniques have been also investigated and employed to increase the stroke of comb-drive actuators. These include shape optimization of the comb fingers [29], [30], varying comb-finger lengths [12] within a comb, multiple comb sets that are sequentially powered [31], [32], and flexure-based displacement amplifiers [33]–[36]. While these techniques have their respective pros

7 =0, 100mn

0.05

0.025

 $10 \times 10^{5}$ 

K<sub>x</sub> (Normalized)

2

0

0

Fig. 9. Motion direction stiffness for different values of  $L_3$  in the C-DP-DP flexure [(solid line) closed-form analysis; (markers) FEA].  $K_y$  stiffness is normalized with respect to  $EI_1/L_1^3$ .

and cons, they are peripheral to the present discussion and therefore not considered in further detail here.

# **III. PROPOSED DESIGN**

To achieve the desired stiffness and error motion goals, our strategy is to work with the DP-DP flexure but constrain its secondary stages in a unique manner. In the proposed C-DP-DP flexure mechanism [37], as shown in Fig. 2, the two secondary stages are connected to an external clamp via secondary P flexures. The high rotational stiffness of these P flexures minimizes any relative Y displacement between the two secondary stages, forcing them to maintain Y/2 displacement at all times. This constrains these stages from responding to an X-direction force on the motion stage. In addition, the low X-direction stiffness of the secondary P flexures offers minimal resistance to the kinematic X-direction displacement of the secondary stages. The analytical relations for the motion and bearing stiffness of the C-DP-DP flexure have been separately derived [38] and are summarized here

$$K_y = \frac{EI_1}{L_1^3} \left( 2k_{11}^{(0)} + \frac{9k_{11}^{(0)}k_{11}^{(1)}}{20} \left(\frac{L_1^3}{L_3^3}\right) \left(\frac{Y}{L_1}\right)^2 \right)$$
(14)

$$K_{x} = \frac{EI_{1}}{L_{1}^{3}} \frac{2k_{33}}{\left(1 + k_{33} \left(g_{11}^{(1)} + \frac{\left(k_{11}^{(1)}\right)^{2}}{k_{11}^{(0)}(1+\eta)}\right) \left(\frac{Y}{2L_{1}}\right)^{2}\right)} \quad (15)$$
where  $\eta = \left(\frac{6W_{3}^{2}L_{1}^{3}}{k_{11}^{(0)}L_{2}^{2}L_{3}T_{3}^{2}}\right).$ 

The beam characteristic coefficients here are the same as earlier [see (9)]. Although beams in the secondary parallelograms are assumed to be of uniform thickness (i.e.,  $a_o = 0.5$ ), it is straightforward to introduce a separate beam shape parameter for them, if needed.

Equation (14) shows that there is a slight increase in  $K_y$  because of the external clamp. However, this can be mitigated by choosing a large-enough secondary parallelogram beam length  $L_3$  (see Fig. 9). Next, the effectiveness of the clamp in restricting the precipitous drop in the bearing direction stiffness  $K_x$  is captured via the dimensionless parameter  $\eta$  in (15). The



0.075

Y/L

0.1

0.125

0.15

external clamp prevents relative Y motion between the two secondary stages by employing the high rotational stiffness of its constituent parallelograms, which manifests itself in the form of  $\eta$ . For low values of  $W_3$ , this stiffness and therefore  $\eta$  are small. It may be analytically seen that as  $\eta \rightarrow 0$ , the  $K_x$  stiffness becomes exactly the same as that for a DP-DP flexure, which corresponds to a completely ineffective clamp (see Fig. 10). However, as  $W_3$  increases, parameter  $\eta$  also increases. Equation (15) shows that as  $\eta$  becomes very large, the kinematic term  $(k_{11}^{(1)})^2/k_{11}^{(0)}$  vanishes and only the elastokinematic term  $g_{11}^{(1)}$  remains. Since the latter is at least two orders of magnitude smaller than the former, this implies that the drop in  $K_x$  stiffness now is significantly reduced and that the clamp proves to be effective. Furthermore, because of the sensitivity of the elastokinematic term to beam shape, reinforced beams  $(a_0 < 0.5)$  may be used to produce even greater improvements in the bearing stiffness  $K_x$  of the C-DP-DP flexure (see Fig. 6). This is in contrast with the DP-DP flexure for which beam reinforcement produces marginal benefits.

The  $(K_x/K_y)$  stiffness ratio of the C-DP-DP flexure for two different values of  $a_o$  is shown in Fig. 6, which demonstrates its superior stiffness characteristics compared with the DP-DP and P flexures. Moreover, unlike the P flexure, the C-DP-DP flexure provides inherently zero  $E_x$  and  $E_\theta$  error motions. Because of these attributes, it is evident in Fig. 6 that the C-DP-DP is capable of producing very large strokes in comb-drive actuators.

The in-plane yaw stiffness  $K_{\theta}$  of a C-DP-DP flexure with high  $\eta$  remains the same as that for the DP-DP, as given by (12). As aforementioned, this stiffness can be independently made large by appropriate choice of the dimensions  $W_1$  and  $W_2$  such that it does not affect the snap-in condition and therefore the actuation stroke.

All the closed-form analytical results in this section assume perfectly rigid stages, external clamp, ground anchors, and beam reinforcements. These results have been also validated via nonlinear finite-elements analysis (FEA). A comparison between the closed-form and FEA results is presented in Figs. 9 and 10 for the normalized  $K_y$  and  $K_x$  stiffness values, respectively.



The following dimensions were used to generate the results plotted in these figures:  $L_1 = 1 \text{ mm}$ ,  $T_1 = 5 \mu \text{m}$ ,  $W_1 = 250 \mu \text{m}$ ,  $W_2 = 400 \mu \text{m}$ ,  $L_2 = 1.1 \text{ mm}$ ,  $a_0 = 0.5$ , and  $H_1 = 50 \mu \text{m}$ ;  $W_3 = 250 \mu \text{m}$ , and  $L_3$  varies in Fig. 9; both  $L_3$  and  $W_3$  vary in Fig. 10.

## **IV. COMB-DRIVE ACTUATOR DESIGN RECIPE**

In this section, we present a systematic procedure for designing a C-DP-DP flexure-based comb-drive actuator to maximize its actuation stroke while minimizing device footprint and actuation voltage. In an actual application, one would take into account any additional constraints imposed by that application in addition to the procedure and steps presented here.

Given the several constraints and tradeoffs involved, the goal here is to obtain a good starting point for the flexure and comb-drive dimensions (i.e.,  $L_1$ ,  $T_1$ ,  $a_0$ ,  $W_1$ ,  $W_2$ ,  $L_2$ ,  $L_3$ ,  $W_3$ , G,  $L_f$ , and  $T_f$ ) based on some simplifying assumptions and subsequently iterate to further refine the overall design.

The following assumptions are initially made and are revisited during later design steps.

Analytical results in the previous section show that the optimization of the external clamp and secondary parallelograms is decoupled from the final stiffness of the C-DP-DP flexure, as long as the clamp is effective. Since an optimal clamp that provides high η can always be created subsequently, motion and bearing stiffness values corresponding to high η(→∞) are assumed at the onset of this design procedure, i.e.,

$$K_y = \frac{2EI_1k_{11}^{(0)}}{L_1^3} \tag{16}$$

$$K_x = \frac{2EI_1}{L_1^3} \frac{k_{33}}{\left(1 + k_{33}g_{11}^{(1)} \left(\frac{Y}{2L_1}\right)^2\right)}.$$
 (17)

- The motion stage, secondary stage, external clamp, ground anchors, and beam reinforcements are all assumed to be perfectly rigid.
- 3) Since the in-plane rotational stiffness  $K_{\theta}$  of the C-DP-DP flexure can be made independently high, it is assumed large enough to be ignored in the first iteration.
- 4) Although the C-DP-DP flexure exhibits theoretically zero error motions in the X- and Θ-directions because of its inherent symmetry, misalignment and error motions due to fabrication imperfection are inevitable. Therefore, a stability margin of S = 1 is assumed to provide robustness against such nondeterministic factors. This may have to be revised in subsequent iterations.
- 5) Silicon is chosen as the flexure and comb-drive material, which sets an upper bound for the maximum achievable stroke due to mechanical failure. For the C-DP-DP flexure, the yield limit is given by [7]

$$Y_{\text{yield}} \le \left(\frac{8}{k_{11}^{(0)}}\right) \left(\frac{S_y}{E}\right) \left(\frac{L_1^2}{T_1}\right) \tag{18}$$

where  $S_y$  is the material yield strength.

6) An initial comb-finger engagement  $Y_0$  is needed to overcome fringing effects that are important for small Y displacements. However,  $Y_0$  is much smaller than the maximum Y displacement and is therefore initially dropped in the design procedure.

With these assumptions, one can now substitute the motion and bearing direction stiffness expressions for an optimal C-DP-DP [see (16) and (17)] into the snap-in condition (3) for a comb-drive actuator

$$\frac{k_{33}}{k_{11}^{(0)} \left(1 + k_{33} g_{11}^{(1)} \left(\frac{Y_{\text{max}}}{2L_1}\right)^2\right)} = \frac{4Y_{\text{max}}^2}{G^2}.$$
 (19)

This snap-in condition corresponds to maximum motion direction displacement  $Y_{\text{max}}$ , which is also the actuation stroke, and associated stiffness values. The actuation voltage  $V_{\text{max}}$  at this maximum displacement may be obtained from (1), i.e.,

$$NV_{\rm max}^2 = \frac{E}{6} \left(\frac{T_1}{L_1}\right)^3 \frac{k_{11}^{(0)} Y_{\rm max} G}{\varepsilon}.$$
 (20)

The left-hand side in the above equation represents the actuation effort, which should generally be minimized. Lower N helps reduce the device footprint, and  $V_{\max}$  is often restricted by practical instrumentation and operational limits.

As aforementioned,  $k_{11}^{(0)}$ ,  $g_{11}^{(1)}$ , and  $k_{33}$  are all functions of  $a_o$ and  $(T_1/L_1)$ , as given by (9). We next present a step-by-step recipe for choosing the dimensions of the C-DP-DP and comb drive that employs the analytical knowledge compiled so far. Step 1: Start with assuming a dimension for the flexure beam

length  $L_1$ , which directly impacts the device footprint. In the first iteration, we choose an initial  $L_1$  value of 1 mm.

Step 2: Minimizing the  $T_1/L_1$  ratio lowers the actuation effort [see (20)] and reduces the bending stress in the flexure beams. A smaller  $T_1/L_1$  ratio also increases the left-hand side of (19), thus delaying the snap-in condition. Therefore, the flexure beam thickness should be chosen to be a small value dictated by the practical limits of the microfabrication process. As mentioned in the next section, this limit is 1.7  $\mu$ m in our case, and with an adequate safety margin, we choose  $T_1 = 3 \mu$ m, which corresponds to a  $T_1/L_1$  ratio of 0.003.

<u>Step 3:</u> Now, the two key remaining design variables in (19) and (20) are  $a_0$  and G. These two equations may be simultaneously solved to eliminate G to produce:

$$NV_{\rm max}^2 = \frac{E}{3\varepsilon} \left(\frac{T_1}{L_1}\right)^3 Y_{\rm max}^2 \sqrt{\left(k_{11}^{(0)}\right)^3 \left[\frac{1}{k_{33}} + \left(\frac{Y_{\rm max}}{2L}\right)^2 g_{11}^{(1)}\right]}.$$
(21)

The above condition can be plotted on an  $NV_{\text{max}}^2$  versus  $Y_{\text{max}}$  graph for multiple fixed values of  $a_0$  (see Fig. 11). Each solid line, referred to an iso $-a_0$  line, represents a fixed value of  $a_0$  and varying values of G. Similarly, the above equations are solved to eliminate  $a_o$ , and the resulting condition is plotted on the same  $NV_{\text{max}}^2$  versus  $Y_{\text{max}}$  graph for fixed values of G. Each dashed line, referred to an iso-G line, represents a fixed value of G and varying values of  $a_0$ . Moving along an iso $-a_0$  line, it is clear that for a given beam shape  $a_0$ , one can achieve higher stroke by increasing the comb gap G, but this also increases



Fig. 11. Design and performance space for C-DP-DP flexure-based combdrive actuators.

the actuation effort  $NV_{\text{max}}^2$ . Similarly, moving along an iso-G line, one can achieve greater stroke by reducing  $a_0$ , which once again leads to a higher actuation effort.

From the design and performance space presented in Fig. 11, one can graphically choose the beam shape  $a_0$  and comb gap G to maximize stroke while minimizing the actuation effort. There is a lower bound on G dictated by the microfabrication process (2  $\mu$ m, in our case) and an obvious upper bound on  $a_0$  (0.5). These bounds produce a feasible design and performance space, indicated by the shaded region in Fig. 11.

<u>Step 4:</u> At this point, one can either set a maximum allowable actuation effort and pick the corresponding actuation stroke or alternatively choose the desired actuation stroke and pick the actuation effort. We choose a desired actuation stroke greater than or equal to 250  $\mu$ m.

Step 5: For a desired stroke, clearly, smaller values of  $a_o$  and G in the feasible design space result in the lowest actuation effort. However, one has to be cautious while choosing small values of these two design variables. Small  $a_o$  leads to increasingly higher stresses in the flexure beams, and the material failure becomes a concern. For a given  $Y_{\text{max}}$ ,  $a_0$  may be chosen to maintain an adequate margin of safety against material failure using (18). Separately, the snap-in condition becomes highly sensitive to error motions  $E_x$  for very small G, and the assumed safety margin S of 1 can prove to be inadequate. For our final designs, we chose  $a_0 = 0.2$  and G values in the range of 3  $\mu$ m to 6  $\mu$ m.

<u>Step 6:</u> Having chosen  $a_o$  and G in the previous step, we now have a numerical value of the actuation effort from Fig. 11. The number of comb fingers N can be chosen next while keeping the maximum actuation voltage  $V_{\rm max}$  within relevant practical limits. We selected  $V_{\rm max} = 150$  V based on our existing instrumentation capabilities.

<u>Step 7:</u> One can now start to lay out the flexure mechanism and the comb drive. The dimensions  $W_1$  and  $W_2$  should be chosen such that rotational stiffness  $K_{\theta}$  given by (12) is adequately high. If it is more than an order of magnitude higher than  $K_x L_4^2$  at  $Y_{\text{max}}$ , the contribution of the rotational stiffness can be entirely ignored. Here,  $L_4$  is the distance along the Y-axis from the center of the flexure mechanism to the tip of the



Fig. 12. Reduction in predicted actuation stroke in the presence of finite rotational stiffness.

comb fingers. If this condition is not met, then the rotational stiffness should be taken into account in the next iteration using a modified version of the snap-in condition [10]. The ratio  $\gamma$  between the reduced actuation stroke due to finite rotational stiffness and the predicted stroke assuming infinite rotational stiffness is graphically illustrated for two values of  $a_0$  in Fig. 12. Step 8: The external clamp is designed next. Using (14), choose  $L_3$  to be large enough so that the increase in the motion direction stiffness  $K_y$  due to the clamp over the above-selected  $Y_{\text{max}}$  is within a few percent (i.e., 5%, in our designs). Next, choose  $T_3$  to be equal to  $T_1$ . Next, choose  $W_3$  and  $L_2$  to make the clamp effectiveness parameter  $\eta$  at least 100. With an effective clamp thus designed, it can now be included in the overall device layout.

<u>Step 9:</u> Next, choose the in-plane thickness of the reinforced beam section to be at least five times the thickness of the end segment  $T_1$ , which ensures more than 125 times bending stiffness. Furthermore, choose the in-plane thickness for the motion stage, secondary stages, and external clamp to be at least 40 times  $T_1$ . If these dimensions are chosen to be less for whatever reason, then the contribution of these stages to the bearing direction stiffness  $K_x$  should be estimated, and a revised snapin condition with this reduced effective  $K_x$  should be used in the next design iteration.

<u>Step 10</u>: Choose  $Y_0$  to be at least two to three times the comb gap G [39]. Then, choose the comb-finger length  $L_f$  to be slightly greater than  $(Y_0 + Y_{\text{max}})$ . For this comb-finger length, the comb-finger thickness  $T_f$  should be chosen to avoid local snap-in of individual fingers. This is given by the following condition [40]:

$$T_f \ge \left(2\frac{\varepsilon L_f^4}{EG^3}V_{\max}^2\right)^{\frac{1}{3}}.$$
(22)

A margin of safety may be included (i.e., 1.5, in our case) while using the above relation.

<u>Step 11</u>: The depth of the flexure beams and comb fingers, i.e.,  $H_f$ , does not play a role in the overall actuator performance and should be selected to be large enough to avoid out-of-plane collapse during fabrication or operation.

Flexure Design	G	$T_f$	N	$L_{f}$	$T_I$	$L_l$	<i>L</i> <sub>2</sub>	$L_3$	<i>W</i> <sub>1</sub>	<i>W</i> <sub>2</sub>	<i>W</i> <sub>3</sub>	$H_{I}$	$a_{\theta}$	Predicted/Designed Stroke			Measured	Voltage at
														S=0	S=1	S=1.5	SITORE	SIFURE (V)
DP-DP	3	6	50	150	4	1000			400	250		50	0.5	59	50	47	50	85
C-DP-DP	3	9	80	210	4	1000	1000	430	790	250	240	50	0.2	228	191	181	141	95
C-DP-DP	3	7	80	180	4	1000	1000	430	790	250	240	50	0.5	164	138	130	119	90
C-DP-DP	4	8	70	240	3	1000	1000	430	810	250	240	50	0.2	272	230	218	215	104
C-DP-DP	4	7	70	210	3	1000	1000	430	790	250	240	50	0.3	224	188	178	170	91
C-DP-DP	4	7	70	190	3	1000	1000	430	780	250	240	50	0.4	202	169	160	157	91
C-DP-DP	6	8	100	290	3	1000	1500	430	875	300	360	50	0.2	341	287	271	245	119

TABLE I Comb-Drive Actuators That Were Designed, Fabricated, and Tested. All Length Dimensions Are in Micrometers

This concludes the first iteration of the overall actuator's dimensional layout, including the flexure and the comb drive. If the device footprint turns out to be too large or too small, one can start the process again from Step 1 with a different value of beam length  $L_1$ . Once an acceptable footprint is achieved, a final check on the actuation, snap-in, and material failure conditions should be performed while removing the previously listed assumptions. Specifically, the choice of stability margin S should be dictated by the selected comb-gap value G and the accuracy of the microfabrication process used. Accordingly, a higher value of S may be used in subsequent iterations. These iterations can lead to further refinement of the flexure and comb-drive dimensions.

This procedure was used to design several devices that were then fabricated and tested. Young's modulus value of 165 GPa (corrected by 3% for P-type doping), as reported in the literature [41], was assumed. Table I summarizes the dimensional details of the designs that were fabricated, and their testing is discussed in the next section.

Finally, if the comb-drive actuator is designed for an application that requires additional force capability at the motion stage (or shuttle), then the actuation and stability conditions (1) and (2) slightly change. Specifically, the desired additional force should be added to the left-hand side of the **Y**-direction force equilibrium [see (1)]. The rest of the design process can be calibrated and carried out in a manner analogous to the one described above.

### V. EXPERIMENTAL RESULTS AND DISCUSSION

# A. Fabrication

The comb-drive actuators were fabricated using silicon-oninsulator wafers with a device layer of 50  $\mu$ m, a buried oxide layer of 2  $\mu$ m, and a silicon handle layer of 350  $\mu$ m. The device layer is heavily boron doped (P-type) with resistivity less than 0.01  $\Omega \cdot$  cm. The silicon handle layer was first patterned and etched by deep reactive-ion etching (DRIE), and then the buried oxide was removed by HF (49%). Finally, the device layer was patterned and etched by DRIE. After the last etching process, critical point drying was used to dry the samples to avoid stiction [42]. In this drying method, liquid CO<sub>2</sub> is transferred to vapor via the supercritical phase (at  $T_c = 38^\circ$ ,  $P_c = 80$  atm) to avoid capillary forces that arise from the surface tension at the liquid–vapor interface. A scanning electron microscope (SEM) image of a representative fabricated device is shown in Fig. 13.



Fig. 13. SEM image of a microfabricated comb-drive actuator employing the C-DP-DP flexure.

# B. Characterization

The fabricated comb-drive actuators were driven by a dc voltage source using electrical probes applied to the fixed and moving combs. The voltage was swept from zero to the maximum voltage at snap-in in a ramp profile with 1-V increments. The displacement response of the actuators was observed with a microscope and captured by a charge-coupled-device camera. Subsequently, the displacement of the actuators was quantitatively measured using image-processing software. The *Y* displacement measurements were found to be repeatable within 1  $\mu$ m.

# C. Voltage–Stroke Curves

Fig. 14 demonstrates the displacement versus voltage curves for three different flexure designs, each with a comb gap of 3  $\mu$ m, a beam length of 1 mm, and a beam thickness of 4  $\mu$ m. The figure shows theoretical predictions and experimental measurements. The measured stroke of a conventional DP-DP flexure with these dimensions and  $a_0 = 0.5$  was 50  $\mu$ m. Referring to Table I, this indicates that S = 1 is a reasonable stability margin to use for the DP-DP flexure and the microfabrication process described above. The measured



Fig. 14. Y displacement versus voltage for DP-DP and C-DP-DP flexurebased comb-drive actuators [(solid lines) theoretical; (markers) experimental].



Fig. 15. Y displacement versus voltage for C-DP-DP flexure-based combdrive actuators [(solid lines) theoretical; (markers) experimental].

stroke increases to 119  $\mu$ m for a C-DP-DP flexure with the same dimensions as the previous DP-DP flexure. This stroke was further increased to 141  $\mu$ m by using reinforced beams with  $a_0 = 0.2$  while keeping all other dimensions the same. This stroke is 2.82 times higher than the DP-DP flexure, which highlights the superior performance of the reinforced C-DP-DP flexure compared with the traditional DP-DP flexure. In the last two cases, Table I indicates that S = 1 or 1.5 may not be an adequate stability margin when G is small (3  $\mu$ m) and the actuation voltage is relatively large (> 90 V). This is explained by (2), which shows that the critical stiffness in the presence of an error motion or misalignment  $E_x$  depends on the ratio  $E_x/G$ . For a smaller gap G, the effect of any misalignment is more pronounced, thereby necessitating a larger stability margin.

As previously shown, the stroke of a comb-drive actuator can be further improved by increasing the comb gap. This is demonstrated in Fig. 15, where a stroke of 215  $\mu$ m was obtained using a C-DP-DP flexure with a beam length of 1 mm, a comb gap of 4  $\mu$ m, and  $a_0 = 0.2$ . The benefit of beam reinforcement is also evident here. With an identical design, the experimentally measured strokes are 170 and 157  $\mu$ m for  $a_0 = 0.3$  and  $a_0 =$  0.4, respectively. Finally, a large stroke of 245  $\mu$ m at 120 V for a C-DP-DP flexure with a beam length of 1 mm,  $a_0 = 0.2$ , and a comb gap of 6  $\mu$ m is also reported on this figure.

In the last four designs, given the larger comb gap, a stability margin of S = 1 or 1.5 appears to be adequate. In fact, an even smaller stability margin may be also considered because, in these cases, the experimentally measured stroke was limited due to a **Y**-direction pull-in as opposed to the **X**-direction snapin considered in the design procedure.

In terms of the Y displacement versus actuation voltage, the theoretically predicted and experimental results for these seven cases agree within 4%, 31%, 11%, 18%, 6%, 4%, and 6%, in the order that they are presented above. Except for the one outlier (31%), these are within the expected range of deviation given the multiple sources of variability and uncertainty in dimensions and material properties in microfabricated devices.

## D. Performance Comparison

While a direct comparison of the actuation stroke with previously reported comb-drive actuators is tricky because of the many variables (beam length, comb gap, maximum voltage, device footprint, etc.) and specific application-based constraints involved, we present a simple analytical basis that provides some level of comparison.

At large displacements, the first term in the square brackets in (21) is dominated by the second term. Therefore, if this first term is dropped, (21) can we rewritten as

$$Y_{\rm max}^3 = \left(\frac{\varepsilon}{E}\right) \left(\frac{6}{T_1^3} \cdot \sqrt{\frac{2}{\left(k_{11}^{(0)}\right)^3 g_{11}^{(1)}(1+S)}}\right) \left(NV_{\rm max}^2\right) \left(L_1^4\right).$$
(23)

Here, S is no longer assumed to be 1, as in (21); instead, it is retained as a function of the error motions  $E_x$ , as given in (3). The first term on the right-hand side above comprises physical constants, which remain invariant. The second term depends exclusively on the stiffness and error motion characteristics of the flexure mechanism used. The third term represents actuation effort and device footprint (due to N) and should therefore be minimized. The fourth term represents the overall device footprint and should be also minimized.

It can be separately shown that, for most flexure mechanisms based on the parallelogram module (e.g., P, DP-DP, and C-DP-DP) that are used in comb-drive actuation, a relation analogous to (23) can be derived. Everything else remains the same except the second term, which depends on the flexure mechanism design. This shows that, to achieve large stroke while minimizing actuation voltage and device footprint, this second term should be maximized via an appropriate flexure mechanism design. Therefore, a comparison between comb-drive actuators, which highlights the performance of the flexure mechanism used, can be conducted by plotting  $Y_{\text{max}}$  against  $(NV_{\text{max}}^2 L_1^4)^{1/3}$ . This is done in Fig. 16, which illustrates the large stroke capability of the C-DP-DP flexure.

Another advantage of the proposed C-DP-DP flexure design is that, unlike the prebent DP-DP, it provides a  $K_x/K_y$ 



Fig. 16. Comparison of this paper's results with previously reported combdrive actuator designs (V and  $L_1$  are in volts and millimeters, respectively.) The top-left corner corresponds to large stroke, small device footprint, and small actuation voltage.

stiffness profile that is symmetric with respect to the Y = 0 displacement position. This allows bidirectional actuation and, therefore, twice the actuation stroke (~ 500  $\mu$ m) for approximately the same device footprint and actuation voltage. For unidirectional operation, the stroke can be further improved by prebending the beams of a C-DP-DP flexure. Unlike the DP-DP case [see (13)], here, the maximum prebend limit is much higher while maintaining S > 1

$$Y_{P-B-\max} \approx \sqrt{\frac{4GL_1}{\sqrt{k_{11}^{(0)}g_{11}^{(1)}}}}.$$
 (24)

For typical dimensions ( $L_1 = 1 \text{ mm}, G = 3 \mu \text{m}, a_0 = 0.2$ ), this allowable prebend is as large as 400  $\mu$ m, which in theory could lead to a Y displacement of approximately 500  $\mu$ m at snap-in. Thus, ultimately, the flexure and the comb drive may be designed such that the actuation stroke is limited by the material failure criteria or the available actuation voltage, instead of sideways snap-in instability.

## VI. CONCLUSION

There are four main contributions in this paper. Foremost, it has presented a novel C-DP-DP flexure mechanism with reinforced beams that offers high bearing and rotational direction stiffness ( $K_x$  and  $K_{\theta}$ ), low motion direction stiffness ( $K_y$ ), and zero error motions ( $E_x$  and  $E_\theta$ ), all over a large range of motion direction displacement. Second, closed-form analytical expressions for these nonlinear stiffness characteristics have been provided that capture their parametric dependence on the flexure dimensions and the beam shape  $(a_0)$ . Third, it has been shown that this flexure mechanism helps delay the onset of snap-in instability in comb-drive actuators to provide greater actuation stroke. A systematic step-by-step procedure for designing a C-DP-DP flexure-based comb-drive actuator to maximize its actuation stroke while minimizing device footprint and actuation voltage has been presented. Finally, several actuators have been microfabricated and experimentally tested

to demonstrate very large actuation strokes. For 1-mm flexure beam length and  $6\mu$ m comb gap, a stroke of 245  $\mu$ m is reported at 120 V. This experimental proof, along with the analytical formulation, clearly highlights the effectiveness of the C-DP-DP flexure for large-range comb-drive actuators. These results present the potential for eliminating sideways instability as the dominant effect that has so far limited the actuation stroke in electrostatic comb drives.

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